Ropelength of braids and tangle decomposition of knots
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Abstract

This paper looked into the upperbounds for the ropelength of knots and links. A new method, the cylinder embedding method, was used to calculate ropelength of twists. This method was then extended into efficently constructing integral tangles to minimize the ropelength. This construction showed the initial length of one crossing in B_3 to be linear in terms of the number of crossings of σ_i. Then used on algebraic tangles, it also shows the ropelength is linear in the crossing number.

1 Introduction

Knot Theory is an area of mathematics which is used in topics related to topology, especially in fields of biology and physics. The specific area of knot theory which looks at braids has been most fruitful in upper and lower bounds of the ropelength of the knot or braid. In this paper, the construction of intregral twists expanded into the exploration of ropelength of braids, then algebraic knots and links. Looking at these specific types of knots were important due to the connections that could be made between ropelength and crossing number of the knots. Previous studies have shown progression of upper and lower bounds of ropelength of a knot, K, denoted $L(K)$. Researcher [2] has shown a lower bound of $O((Cr(K))^{3/4}$, where $Cr(K)$ is the crossing number of knot K. While [4] has shown the upper bound to be $O((Cr(K))^{6/5}$. The goal of this paper is to show how ropelength of certain closed links and knots creates a linear upper bound in terms of crossing number.

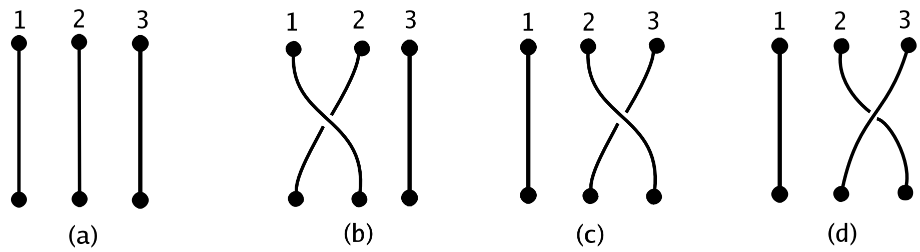
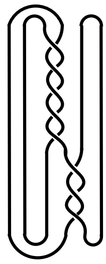


Figure 1: The generators of B_3 are as follows. (a) e: Trivial braid with no crossings. (b) σ_1: Braid with the crossing of strand one over two. (c) σ_2: Braid with the crossing of strand two over three. (d) σ_2^{-1} : Braid with the crossing of strand three over two.

2 The Braid Group

The braid group is denoted B_n, where n is the number of strings in the braid. The strings are arranged in such a way where they are fixed in place and travel downward with crossings moving horizontally, left to right. Each crossing of strand i over strand $i+1$ is denoted by σ_i (Figure 1b,c). The inverse of σ_i, denoted σ_i^{-1}, is the crossing of strand $i+1$ over strand i, (Figure 1d).
B_n is group with each σ_i as a generator, where $i = 1, ..., n-1$. The trivial braid, e, of B_n, is a braid with no crossings (Figure 1a). Multiplication of generators is the compostition of those generators. The second generator in the compostion starts at the points where the first generator ended. The composition of σ_i and σ_i^{-1} is the trivial braid, $\sigma_i\sigma_i^{-1} = e$.

When the same generator is used consecutively more than once, it is denoted σ_i^a, where a is the numer of crossings. For example, four consecutive crossings of σ_1 is denoted σ_1^4. Therefore, the composition of three crossings of σ_1, one crossing of σ_2 and then two crossings of σ_1 would be written as $\sigma_1^3\sigma_2\sigma_1^2$, which is called the braid word. For any braid, β, the braid length, $l(\beta)$, is the number of generators in the braid word or the sum of all the absolute values of $a's$. So in the previous example, $l(\beta) = 6$. In general, for $\beta = \sigma_1^{a_1}\sigma_2^{a_2}...\sigma_i^{a_q}, l(\beta) = |a_1| + |a_2| + ... + |a_q|$.
For a braid, $\beta \in B_n$, the closed braid, denoted β_c, is formed by connecting the begining vertices with the ending vertices without introducing any new crossings.



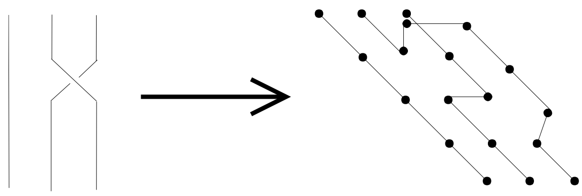
3 Ropelength

Think of K being made of rope with radius r. The ropelength of K, $L(K)$, is the ratio of the length of the rope and the rope's radius; $L(K) = \frac{L}{r}$ [7].
An alternate way to look at the ropelength is finding the thickest smooth knot without intersections. In [6], theorem 1 proves the requriements necessary to ensure the ropes do not intersect.

- The distance between doubly-critical points must be at least twice the radius.
- The radius of curvature must be at least the radius of the rope.

3.1 Lattice Embedding Approach

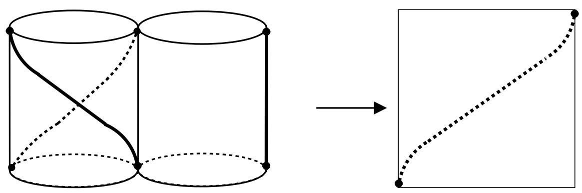
In [4], Diao and Ernst embedded braids on a 3-deminsional lattice. So a three string braid would with one crossing would be shown as follows:



To calculate the ropelength, at each change of direction (right angles in the lattice) an arc of a quarter circle was inscribed from the midpoint to one segment to the midpoint of the next segment, creating a arclength of $\pi/4$ for each right angle. By adding up arclengths along with the straight pieces and using their radius of 1/2 (which fulfills the previously mentioned requirements since on a lattice points are 1 unit apart) 2 strings with one crossing would have a ropelength of $3\pi + 12$, with an additional 8 for every string not included in the crossing. In B_n, $L(\sigma_i) = 3\pi + 12 + 8(n-2)$. Therefore in $B_3, L(\sigma_i) \approx 29.42$.

3.2 Cylinder Embedding Approach

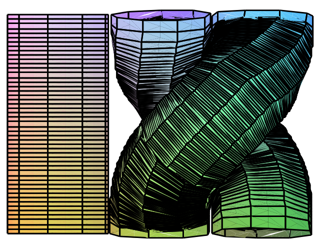
Moran's method, [7], of finding the ropelength of the twists of a pretzel knot used the path of a double helix. This method did not allow for proper blocks that could be stacked to create a braid. Therefore we construct a single block for each generator, σ_i, that can be stacked on top of one another. Visualizing the core of each rope as a string, the strings can be embedded on cylinder of radius 1 to ensure the distance between strings would remain 2 units. The string also has to have a vertical tangent vector at the begining and end of the block to ensure that the blocks can stack directly on top of each other, lining up the strings. Inbetween the two arcs, the string has a slope 1 to ensure the proper requirements so the strings do not intersect and for ease of calcualtions. Once the string has a path on the cylinder, the cylinder can be cut in half so the string can lay flat. Since the flatted half-cylinder preserves arclength, it can be used to find a function for the string's path.



The function is split into three pieces to determine the appropriate curve. The function is broken down as follows:

$$f(t) = \begin{cases} arcsin(t-1) + \frac{\pi}{2} & 0 < t \leq 1 \\ t + \frac{\pi}{2} - 1 & 1 < t \leq \pi - 1 \\ arcsin(t-(\pi-1)) + \frac{3\pi}{2} - 2 & \pi - 1 < t \leq \pi \end{cases}$$

The functions then paramertized back onto the cylinder in the form $[cos(t), sin(t), f(t)]$.



The ropelength for one-string with radius one and creating one crossing is approximately $3.82 + \sqrt{2}(\pi - 2)$. Since the height of our twists are $2\pi - 2$, the appropriate ropelength would be added on for every string not included in the crossing. In $B_n, L(\sigma_i) = 2(3.82 + \sqrt{2}(\pi - 2)) + (2\pi - 2)(n - 2)$. Therefore in $B_3, L(\sigma_i) \approx 15.17$

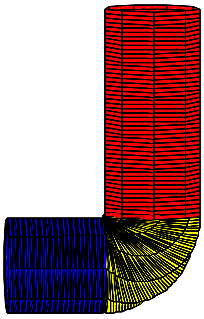
When two strings have more than one crossing in a row, the function changes due to the fact that we do not have to stack in between crossings, the twist can continue to flow. The ends stay as the same arcsine curve while the middle line extends in length.

Lemma 1. *In B_3 with σ_i^a , the piecewise fuction is as follows:*

$$f(t) = \begin{cases} arcsin(t-1) + \frac{\pi}{2} & 0 < t \leq 1 \\ t + \frac{\pi}{2} - 1 & 1 < t \leq a\pi - 1 \\ arcsin(t - (a\pi - 1)) + (a+1)\pi - \frac{\pi}{2} - 2 & a\pi - 1 < t \leq a\pi \end{cases}$$

Thus, $L(\sigma_i^a) \leq (5.64 - 4\sqrt{2} + \pi) + a(2\pi\sqrt{2} + \pi)$.

Now, lemma 1 can be used to find an upper bound for the ropelength of a closed braid. To use the least ropelength when creating a closed braid, the braid word is split into two equal pieces which are placed back to back and connected at the ends. First the pieces need to be spaced properly to ensure no overlapping of the rope. Since the rope has radius of one, once the ropes are forced away from each other during the crossings they create a distance of 4 units across the widest part of the twist. Therefore the two pieces of the braid will have four units between them. Then, a quarter circle is added to the ends of the strings to connect with a horizontal tube between the two pieces. The quarter circle will bring the string directly above the outer edge of the rope (a radius of one), thus only two units are need for the horizontal tube since each rope will contribute the radius of one from the quarter tube at the end of each rope.



The ropelength of the quarter tube is $\pi/2$ and the horizontal tube 2. Thus to connect two pieces of the braid at the end, $L(connector) = \pi/2 + 2 + \pi/2 = \pi + 2$. Let C=all connectors, so in B_3, the closed braid with the six connectors, $L(C) = 6(\pi + 2) = 6\pi + 12$. When the braid word is made up of an odd number of generators, then one trivial braid will be added to make two pieces of the braid equal length. Thus in $B_3, L(e) = 3(2\pi - 2) = 6\pi - 6$. These pieces can then be used to show the following theorem.

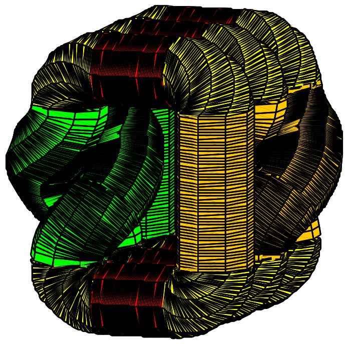
Theorem 1. *For $\beta \in B_3, \beta = \sigma_1^{a_1}\sigma_2^{a_2}...\sigma_i^{a_q}$.*
If $l(\beta)$ is even, then

$$L(\beta_c) \leq q(5.64 - 4\sqrt{2} + \pi) + l(\beta)(2\pi\sqrt{2} + \pi) + [6\pi + 12].$$

If $l(\beta)$ is odd,

$$L(\beta_c) \leq q(5.64 - 4\sqrt{2} + \pi) + l(\beta)(2\pi\sqrt{2} + \pi) + [12\pi + 6].$$

Proof. By construction of the previously mentioned pieces. □



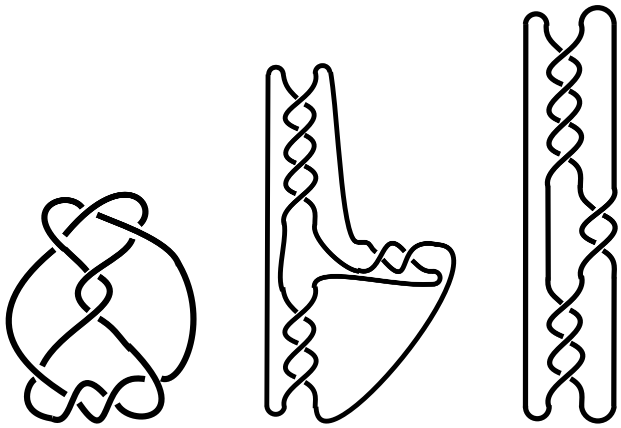
4 Conway Notation

The Conway Notation is a method that is used to describe prime knots by considering tangles within a knot. A tangle is a section of a projection of a knot that consists of the building blocks for the knot. An integral tangle will consist of the consecutive twisting of two strands. For further reference to the multiplication and addition of tangles see [1]. A knot constructed using only multiplictaion or addition will be denoted K_m and K_a, respectively. For a knot K that uses both multiplication and addition of tangles, the subscript will be omitted.

4.1 Tangle Multiplication

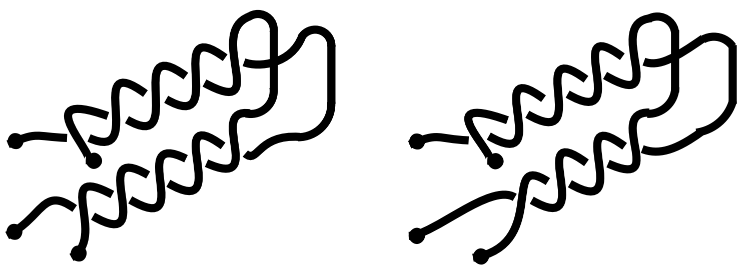
Using the Conway notation for tangles, a knot, K_m, can have the projection similar to a braid in B_3 by changing the isotopic view (Figure 2). Note that K_m is always a 2-bridge link, which is known to be alternating. The braid is split up into sections for each integral tangles, which become the folded sections (Figure 3), and sections for the straight pieces, which become the connector sections, denoted C for all the connectors used in a knot. Let $K_m = a_1 a_2 ... a_q$ where each a_i denotes the number of positive twists in the tangle. Also, q denotes the number of integral tangles. Note that $Cr(K) = \sum_{i=1}^{q} a_i$ therefore, $q \leq Cr(K)$. For each integral a_i, let F_{a_i} denote the ropelength of the set of folded twists. The sum of all the F_{a_i} will be denoted F_{total}.

Lemma 2. *In K_m, $F_{total} \leq O(Cr(K_m))$.*



(a) (b) (c)

Figure 2: (a) Conway tangle 423. (b) Tangle 423 with changed isotopic view to show tangle multiplication. (c) Tangle 423 in braid form.

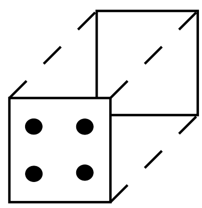


(a) (b)

Figure 3: (a) Example of σ_i^{10} folded. (b) Example of σ_i^{9} folded with trivial on end of twists.

Proof. The following pieces are used to calculate the ropelength.
Let $t = \lceil \frac{a_i}{2} \rceil$ and $b = a_i - t$
b_1 = set of twists for top = $2(3.82 - 2\sqrt{2} + t\pi\sqrt{2})$
b_2 = set of twists for bottom = $2(3.82 - 2\sqrt{2} + b\pi\sqrt{2})$
e = trivial braid of two strands = $4\pi - 4$
c = connectors for two strands = $2\pi + 4$
Therefore, $F_{a_i} = b_1 + b_2 + c + (t-b)e$. When all a_i are even, then $F_{even} \leq q(15.28 - 8\sqrt{2}) + 2\pi + 4 + 2\pi\sqrt{2}\sum_{i=1}^{q} a_i$. When all a_i are odd, then $F_{odd} \geq q(15.28 - 8\sqrt{2}) + 6\pi + 2\pi\sqrt{2}\sum_{i=1}^{q} a_i$. Also, $q \leq Cr(K_m)$ when q is either even or odd. Since F_{total} is in terms of q, it can be said that the ropelength of the constructed folds are linear in the crossing number. □

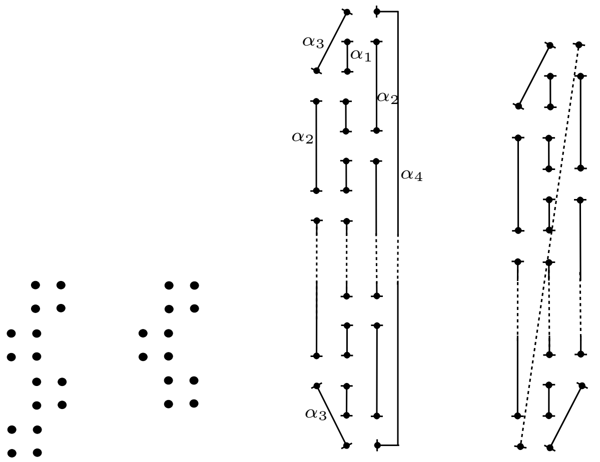
After all the folding, each integral tangle creats a "four-prong circut" that will be used to connect all the folded tangles. For better visual understanding, think of each folded integral tangle in its own drawer where the depth is determined by the length of the individual tangles. The width and height stays constant at 4 units.



Lemma 3. *For K_m, the greatest depth is $\lceil \frac{L(a_i)}{2} \rceil + 2$ for largest $|a_i|$ while the height and width are 4.*

Proof. The depth is determined by how far the longest tangle travels which is the length of the folded tangle. Therefore the tangle is $\lceil \frac{L(a_i)}{2} \rceil$ long plus the 2 units for the connector on the end of the fold. The height and width are unchanged because the tangle never leaves the space between the two planes on which the fold is created. They are both 4 units due to the fact that we have to ensure that the rope does not intersect with other tangles, so for the arrangement of the cores, with a radius of one, in a square it creates a height and a width of 4. □

We now connect the integral tangles in an effecient way to reduce ropelength. There are q drawers lined up vertically that need to be connected to other drawers (Figure 4), then split into two dressers facing each other (Figure 5) 4 units apart to create the necessary space for the connectors. We describe six types of connections that are used. (See figures 4b and 5a for locations of α_i's.)
For q tangles, let $j = \lceil \frac{q}{2} \rceil$ and $k = q - j$.
α_1 : Connects F_{a_i} to $F_{a_{i+1}}$ for $i = 1...j$ and $i = k...(q-1)$, therefore $(q-2)$ connectors.



(a) (b) (c)

Figure 4: (a) Set up of the four-prong when q is even (left) and odd (right). (b) Four pronged circuits with connectors when q is odd. (c) Four pronged circuits with connectors when q is even.

α_1' : Connects F_{a_j} to $F_{a_{j+1}}$.
α_2 : Connects F_{a_i} to $F_{a_{i+2}}$ for $i = 1...(q-2)$ and F_{a_i} to $F_{a_{i+2}}$ for $i = 2...(q-2)$, therefore $q-2$ connectors.
α_2' : Connects F_{a_j} to $F_{a_{j+1}}$ and $F_{a_{j-1}}$ to $F_{a_{j+1}}$.
α_3 : Connects F_{a_1} to F_{a_2} and $F_{a_{q-1}}$ to F_{a_q}.
α_4 : Connects F_{a_1} to F_{a_q}.

Lemma 4. *For K_m, $L(C) \leq O(Cr(K_m))$.*

Proof. To prove this lemma it will be shown how each connector is made and the ropelength of each connector will be calculated. Then the lemma follows from the summation of all the necessary pieces for the knot. In particular, we will show that,
$L(\alpha_1) = \pi$
$L(\alpha_1') = 4$
$L(\alpha_2) = \pi + 4$
$L(\alpha_2') = \pi + 3.66$
$L(\alpha_3) = \pi + 2.47$
$L(\alpha_4) = \pi + 3.66$
For the lengths of each α, the core of the rope is thought of on a cubic lattice. Since the radius of each rope is one, the core must be placed every other point on the lattice. For α_1, (Figure 6a) the ropes are right next to each other so a

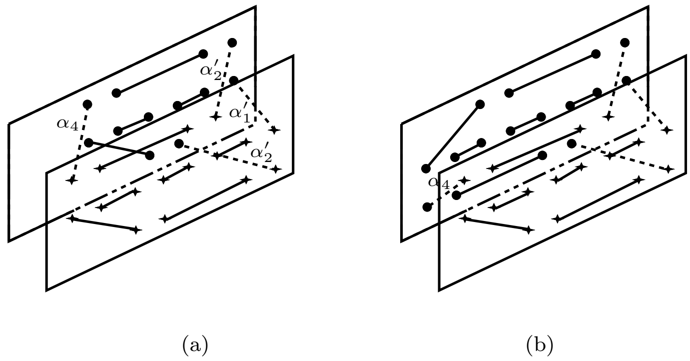


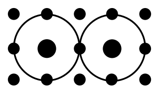
Figure 5: The folded representation of the previously verically alined four-prong circuits. (a) When q is odd. (b) When q is even. The rhombuses represent two planes on which the four-prongs on lined up on. The plane with cicular circuits has tangles into the paper and the plane with star circuits has tangles coming out of the paper. The solid lines are connectors within the plane, while th dashed lines are connectors between the two planes, to create a knot.

half circle having a ropelength of π is all that is needed for any connection of α_1. When connecting the two planes with α_1', since the cores line up directly across from each other, a tube 4 units long is all that is needed because the two planes are 4 units apart. For α_2, the cores are 6 units apart, but once quarter circles are added to the end of the rope, those quarter circles are only 4 units apart (Figure 7a). Therefore the ropelength of the tube is $\pi + 4$. To connect the two planes with α_2', the cores are on a lattice which travel between the two planes (Figure 7b). Since a 45, 45, 90 triangle is created, it is know that the quarter circles come out to the edge of the rope at $(\sqrt{2}/2, \sqrt{2}/2)$ and $(4-\sqrt{2}/2, 4-\sqrt{2}/2)$. Therefore, the distance between the edges of the ropes is found to be approximately 3.66, giving a total length of $\pi + 3.66$. The length of α_3 is found in the same way with the edges of the quarter circles at $(\cos[\arctan(1/2)], \sin[\arctan(1/2)])$ and $(4-\cos[\arctan(1/2)], 2-\sin[\arctan(1/2)])$ (Figure 8a). The distance is approximately 2.47, so $L(\alpha_3) = \pi + 2.47$. The final connector between the two planes is the same as α_2' (Figure 8b), therefore $L(\alpha_4) = \pi + 3.66$ Therefore, $L(C) = 2q\pi + 4q + 5\pi + 11.92$. $\square$

By construction and use of previous lemmas, the following theorem can be shown.

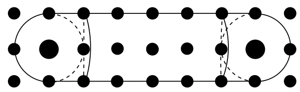
Theorem 2. *For $K_m, L(K_m) \leq O(Cr(K_m))$.*

Proof. For $L(K_m)$, all the pieces need to be added together. Therefore, $L(K_m) = F_{total} + L(C)$. By lemmas 2 and 4, it is shown that F_{total} and $L(C)$ are both linear in the crossing number. Therefore, $L(K_m) \leq O(Cr(K_m))$. $\square$



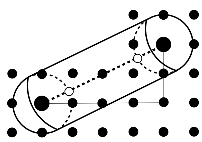
(a) (b)

Figure 6: In the following figures, the large cirlces are the core of the rope in one plane and the stars are the core in another plane while between planes the vertices are square points. (a) α_1 (b) α'_1: square vertex is between planes



(a) (b)

Figure 7: (a) α_2 (b) α'_2: square vertex is between planes

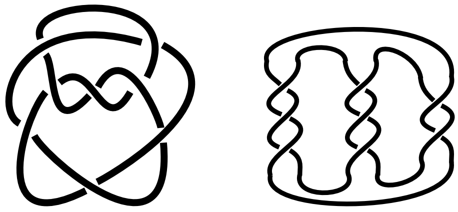


(a) (b)

Figure 8: (a) α_3 (b) α_4: square vertex is between planes

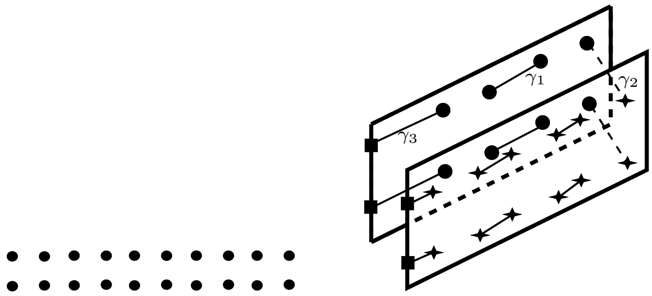
4.2 Tangle Addition

For addition, let $K_a = a_1, a_2, ..., a_q$ where each a_i deonotes the number of possible twists and q denotes the number of integral tangles. The tangles are



(a) (b)

Figure 9: (a) Conway tangle 3,3,2. (b) Tangle 3,3,2 with changed isotopic view to show tangle addition.



(a) (b)

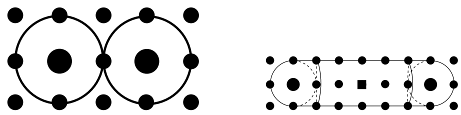
Figure 10: (a) Prongs lined up for tangle addition. (b) Connectors used in tangle addition.

arranged next to each other vertically left to right (Figure 9b). This arranges the four-prong circuits in the same fashion horizontally (Figure 10a). Then the integral tangles are folded and calculated in the same way as in previous section, creating F_{a_i} for $i = 1, ..., q$. Once the four-prong circuits are lined up, it can be folded again so the face each other (Figure 10b), staying 4 units apart to ensure the rope does not intersect.

There are three connectors that will be used, denoted by γ_i. (See 10b for locations of γ_i's.)
For q tangles, let $j = \lceil \frac{q}{2} \rceil$ and $k = q - j$.
γ_1 : connects F_{a_i} to $F_{a_{i+1}}$ twice for $i = 1...j$ and $i = k...(q-1)$, therefore $2(q-2)$ connectors.
γ_2 : connects F_{a_j} to F_{a_k} twice at the end of the fold.
γ_3 : trivial straight rope of 2 strands.

Lemma 5. *For K_a, $L(C) \leq O(Cr(K_a))$.*

Proof. To prove this lemma it will be shown how each connector is made and calculate the ropelength of each connector. In particular, we show
$L(\gamma_1) = \pi$
$L(\gamma_2) = 8$
$L(\gamma_3) = 8$
The construction of γ_1 is the same as α_1 (Figure 11a), therefore $L(\gamma_1) = \pi$. For γ_2, the connector travels straight across between the two planes (Figure 11b), so a rope of 4 units is used for each γ_2. Quarter circles will also be needed at the end of each set of added tangles to create the four-prong to be used in the large braid. Therefore, the length of the end pieces is 2π. From the summation of all the necessary pieces for the knot, $L(K_a) = 2\pi q - 2\pi + 16 + 8(j-k)$ which shows that the ropelength is linear for q. Since $q \leq Cr(K_a)$, the ropelength is also linear for the crossing number. $\square$



(a) (b)

Figure 11: (a) γ_1 (b) γ_2: square vertex is between planes

By construction and use of previous lemmas, the following theorem can be shown.

Theorem 3. *For $K_a, L(K_a) \leq O(Cr(K_a))$.*

Proof. For $L(K_a)$, all the pieces are added together to get $L(K_a) = F_{total} + L(C)$. By lemma 2 and 5, it is shown that F_{total} and $L(C)$ are both linear in the crossing number. Therefore, $L(K_a) \leq O(Cr(K_a))$. $\square$

For every K_a, the integral tangle has its own drawer where the width is determined by the length of a_i while the depth is determined by q. The height is constant again and the four-prong circuits are centered vertically but its location horziontally is determined by the length of a_i.

Lemma 6. *For K_a, the greatest width is $L(a_i) + 8$, the greatest depth is $4(\lceil \frac{q}{2} \rceil)$ while the height is 4.*

Proof. Since the width is determined by the folded a_i, which only goes in one direction, but For the largest a_i, it would extend the width of the drawer by $\lceil \frac{a_i}{2} \rceil$ in one direction. But to compensate for another a_i that is equal or lesser in length, $L(a_i)$ is used to extend the drawer. Also, two units are added onto each end for the connection of the folding of the individual tangles and four for the space between the two planes. Therefore, the width is at most $L(a_i) + 8$. The depth of the drawer is determined by q since the folding method is used (Figure 10). When more a_i's are added together, more four-prongs need to be lined up facing each other. To compensate for an odd q, the ceiling gives the maximum number of four-prong sets on one side. Then since each four-prong set is four units wide and the connectors are kept within the planes, the maximun number of four-prong sets is multiplied by four. Therefore, the depth is at most $4(\lceil \frac{q}{2} \rceil)$. $\square$

4.3 Multiplication and Addition: The Algebraic Tangle

Some Conway tangles have both mulitplication and addition comprised in the knot. For these types of knots, the pieces from the previous sections can be used to create an algebraic tangle, which can be closed to be called an algebraic link. Still, there are some knots that are not algebraic. For further information on algebraic and non-algebraic knots see [1].

When a knot has a Conway notation with both multiplcation and addition, addition is calculated first, followed by multiplication. For example, the tangle, T, $a_1a_2, a_3, a_4a_5a_6, a_7$ could be seen as $T = a_1(a_2, a_3, a_4)a_5(a_6, a_7)$. Here $A_1 = a_2, a_3, a_4$ and $A_2 = a_6, a_7$ would each be treated as new single tangle to be multiplied by the other a_i's. Think of a_1, A_1, A_2 and a_5 each having their own drawer. When calculating the ropelength of the algebraic tangles, the methods of the previous two sections apply. First ropelength is found for each set of addition tangles. By lemmas 2 and 5, $L(A_1) \leq F_{A_1} + 4\pi + 24$ and $L(A_2) \leq F_{A_2} + 2\pi + 16$. Once the addition tangles are constructed, the four-prong ends (Figure 7b, the square vertices) will be used in the four-prong set up for multiplication. The new tangle notation would be $T = a_1A_1a_5A_2$. Once all the drawers are constructed, they are stacked on top of each other then split into two dresser sets. The handles are lined up as four-prongs were and the dressers are spaced 4 units apart, facing each other. Rope can then be used to connect appropriate parts of the handles. From the example of tangle T, one dresser would have a_1 on top of A_1 and the other dresser would have a_5 on top of A_2. The two dressers would be placed facing each other and the appropriate connectors would be used to close the link. The method for multiplication is conintued by using lemmas 2 and 4. Therefore for the example, $L(T) \leq F_{a_1} + L(A_1) + F_{a_5} + L(A_2) + 12\pi + 27.92$.

With this type of construction, it can be shown that:

Theorem 4. *For algebraic knot, K, that admits a minimal crossing algebraic projection, $L(K) \leq O(Cr(K))$.*

Proof. To find the ropelength of a knot or link, folds, connectors and tangles need to be added together. By lemmas 2 and 5, it is shown that the total length of the folds that are added together are linear in the crossing number while lemmas 2 and 4 show how the pieces for connecting folds by multiplication are also linear in the crossing number. Then theorem 2 and 3 show tangles that are multiplied and added together will also be linear in the crossing number. Also, since the tangles are the pieces that make up the knot, we know the crossing number of the tangles are less then or equal to the crossing number of the knot. Hence, for algebraic knot K, $L(K) = F_{total} + L(C)$, where F_{total} and $L(C)$ are both in terms of q and $\sum |a_i|$. Since $q \leq Cr(K)$, then $L(K) \leq O(Cr(K))$. $\square$

After this work was completed, similar conclusions were found in [3] by a different method using Hamiltonian cycles and lattice embedding for constructions of tangles.

5 Further Research

Toward the end of the research, methods to find the ropelength of non-algebraic knots were being looked into. Continuing to look at Conway's theories, the idea of Conway's basic polyhedra was researched and used to discover the best arrangement of the tangles. We obtained partial results by arranging the tangles as a graph with six vertices, each with a degree of 4. Also, by knowing the maximum depth, width, and height of the integral tangles, an maximum upperbound can be found. Research can be continued that looks at better methods of arranging the integral tangles as well as looking at other non-algebraic knots to find uppper bounds for ropelength.

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