Structure of K Braid Satellites Within a Type K Torus

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Abstract

The set of Satellite Knots is one of the three fundemental groups of knots, along with Hyperbolic and Torus Knots. This set is unique in that it contains the set of all composite, or non-prime, knots. In the field of Knot Theory we are mostly concerned with the properties of prime knots, so for my study I focused on the properties of a potential set of prime satellite knots. Specifically we will examine the properties of satellite knots whose companion is a torus of type k and whose braid diagram has an index of k. By deriving braid words from projections of these knots we will examine the structure and properties of these knots and develop a straight forward method for creating knots of this type.

1 Introduction

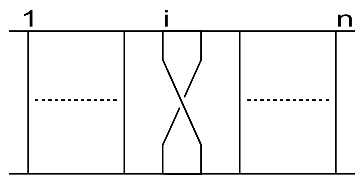
Fundemental to our study is the understanding of several basic ideas within the field of knot theory. The most essential being the idea of the braid, or a series of non-intersecting strands in a cylinder that intersect any given cross section of the cylinder at exactly n points, where n is the number of strands in the braid. It is known that every knot can be described as a braid where the end points of corresponding strands are connected. The smallest number of strands needed to represent a given knot is called the braid index of that knot and factors greatly into our study.

Now to study braids we need to apply a structure that will allow us to manipulate them in a standardized way. For that we define an elementary braid and an algebraic description of the braid group.

Definition 1.1. *The Elementary Braid, σ_i, is any n-stranded braid where the ith stand crosses in front of the $(i+1)$st strand.*

Definition 1.2. *The Algebraic Braid Description: $\mathbb{B}_n$ is the group of braids with index n and is such that*

$$\mathbb{B}_n = < \sigma_i | \sigma_i \sigma_j = \sigma_j \sigma_i ; |i-j| \geq 2, \sigma_i \sigma_{i+1} \sigma_i = \sigma_{i+1} \sigma_i \sigma_{i+1} >$$



The Elementary Braid σ_i

Braids are described using a series of σ_i's to produce braid words that can be manipulated using braid relations.

Example:

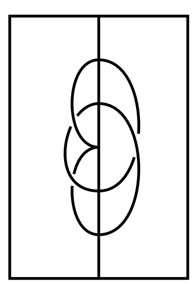
$$\sigma_1 \sigma_3^{-1} \sigma_2 \sigma_1 \sigma_2 = \sigma_3^{-1} \sigma_1^2 \sigma_2 \sigma_1 \in \mathbb{B}_4$$

Now that we understand braids we must describe the space that these braids will exist in. For our purposes it will be benificial to work in $\mathbb{S}^3$ where $\mathbb{S}^3 = \mathbb{R}^3 \cup \{\infty\}$, and use the open-book decomposition of $\mathbb{S}^3$.

Definition 1.3. *The Open-Book Decomposition of $\mathbb{S}^3$ is a descritpion of the space using the coordinate system (r, θ, z). We denote the z-axis as the binding axis and for a fixed value θ the set $H_\theta = \{(r, \theta, z) \in \mathbb{S}^3\}$ is the disk at angle θ with the positive x-axis. As θ ranges from 0 to 2π these disks fill the space. Each of these disk are called a page or fiber of the Open-Book decomposition.*

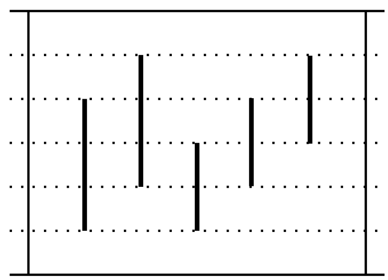
For the most part we will use the Braid axis of our satellite as our binding axis in $\mathbb{S}^3$. Within this space we are able to create a specific type of projection of any given link called the Arc Presentation of the Link.

Definition 1.4. *The Arc Presentation of a link L is an embeding of L in the open-book decomposition such that a finite number of H_θ exists such that each conatain a single continuous arc of L where the end points of these arcs lay on the binding axis.*



An Arc Presentation of the Trefoil using Five Arcs

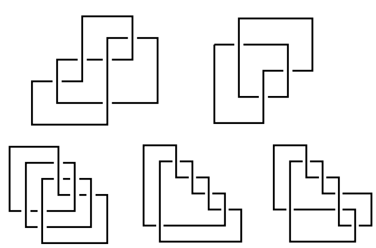
There are several different diagrams that can represent the arc presentation such as the spoke or grid presentation. The spoke presentation represents an "overhead" view of the arc presentation where each "Spoke" is labelled with the end points of the arc it contains. The grid presentation is another version of the arc presentation where each row represents a point on the binding axis, and the segments in each coloumn represnt arcs that span those points. The order of the coloumns from left to right represents the counter clock-wise order of the arcs around the binding axis. A labelled grid presentation assigns a number to each row, or point, and a letter to each segment, or arc.



A Grid Presentaion of the Trefoil

The minimum number of arcs needed to represent a given link in an arc presentation is call the arc index. We can construct arc presentations of knots using what is called a *grid diagram* of a link, as described by Jin and Park[4].

Definition 1.5. *The Grid Diagram of a link is a diagram built using horizontal and vertical segments such that no more than two corners exist in any row or coloumn and the vertical segments cross above the horizontal segments. (It was shown by Cromwell[3] that every Link is isotopic to a Grid Diagram.)*



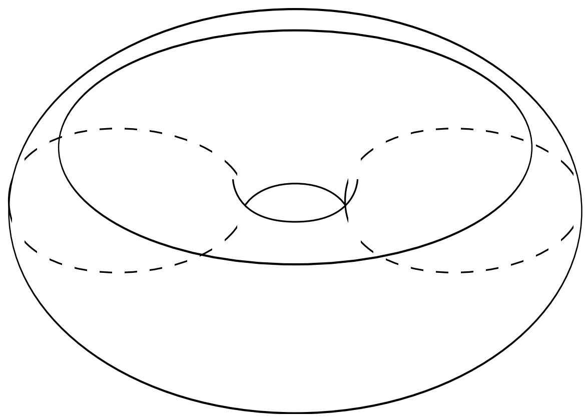
Several Grid Diagrams of Standard Knots

We can construct an arc presentation by pulling the horizontal components horizontally toward the binding axis. It can be observed that the grid diagram relates directly to the grid presentation due to the fact that the horizontal segments of each are projected onto a point on the binding axis to produce the arc presentation. From this we can go directly from the grid diagram to the grid presentation of its arc presentation.

The types of knots under study are the satellite knots and are defined as follows:

Definition 1.6. *A knot K is satellite if it can be completely contained within the interior of a torus T, such that the core of T is isotopic to a non-trivial knot and that K is incompresible in the interior of T.*

To study a satellite we will need to know about the torus it is embedded within. A meridian is an essential loop on the surface of the torus that bounds a disk in the torus. A longitude is a simple loop on the surface of the torus that intersects any meridian once, and is called a preffered longitude if its linking number with the core of the torus—or framing number—is 0.



A Torus with two Meridians and a Preferred Longitude shown.

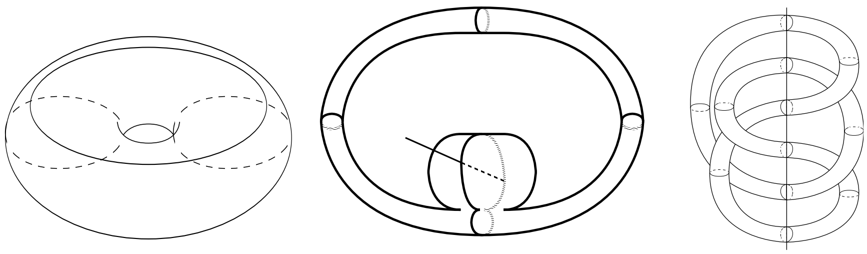
A torus is considered knotted if any preferred longitude is knotted. Birman and Menasco[1] determined three distinct classes of tori in the context of braids.

Definition 1.7. *A Type 0 torus that does not intersect the binding axis and every H_θ intersects the torus along n unique meridian, where n is the braid index of the core of any preferred longitude of the torus.*

Definition 1.8. *A Type 1 torus is such that the surface of the torus intersects the binding axis twice and every H_θ either intersects n disks bound by unique meridian and a disk bound by a meridian and the binding axis or $(n-1)$ disks bound by unique meridian and a disk bound by a meridian and the Binding Axis, where n is the braid index of any preferred longitude of the torus.*

Definition 1.9. *A Type k torus is such that the surface of the torus intersects the Binding Axis $2k$ times and the core of the torus intersects the binding axis k times, where $k \geq 2$.*

The core of the type k torus is isotopic to an arc presentation of the torus. If the arc index of the core of the core of the torus is k then the torus can only be of type l, where $l \geq k$.

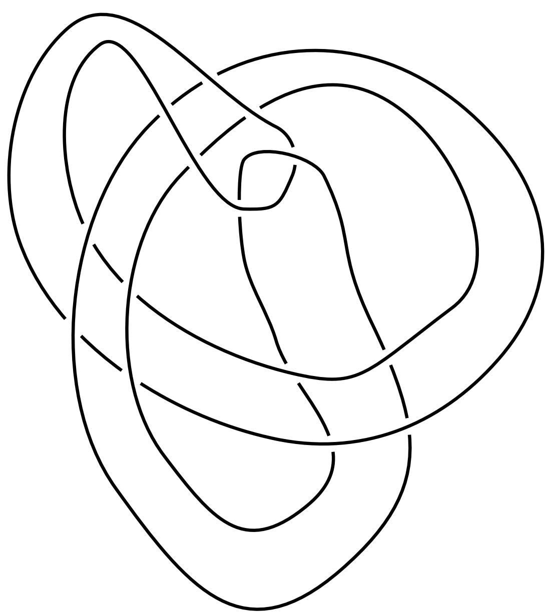


A type 0, 1 and k torus

For each type of torus a satellite is called a proper satellite when the number of times that the satellite intersects any disk bounded by a meridian of the companion, the wrapping number, is greater than or equal to 2.

From Cromwell's text[2] the following is proven:

Theorem 1.1. *A Proper Satellite is prime if its pattern is a prime knot or the trivial knot.*



A proper satellite with an unkot pattern is prime

With this information we will construct, to start, the braid word for a satellite knot with braid index 5 and companion torus being of type 5, and move onto the structure of higher indexed braids.

2 Results

Our Initial interest in the 5 braid with type 5 companion comes from the fact that it is one of two possible structures that could contain prime 5 braid satellites. By constructing the braid words for this type of satellite we will be able to classify the 5 braid prime satellites, and move towards developing a method for constructing satellites whose companion is of type k. By creating a method for constructing these types of satellites we come closer to classifying prime satellites of braid index k.

For the following property of 5 braid satellites we will use three equation developed by Birman and Menasco [1], where b(K) is the braid index of a knot K, w_n is the weight of the strands, as defined below and C is the companion knot.

Definition 2.1.

Type 0 *w is wrapping number of the satellite in the Torus*

Type 1 *Given to types of Disks D and d such that D is bounded by the braid axis and a meridian of the torus and d is bounded by a meridian of the torus, and a specifc H_θ such that H_θ intersects the torus T at 1 disk D and n disks d, where n is the braid index of the core of the torus, then w_0 is the number of strands that intersect H_θ in a disk d and w_1 is the number of strands that intersect H_θ in a disk D.*

Type k *w_n is the number of strands that go around each segment A_n in a Torus T such that $A_1 \cup A_2 \cup \ldots \cup A_n = A \cap T$ where A is the braid axis.*

These equations compute the braid indices of knots within a Type 0, 1 and k Torus respectively.

Type 0: $b(K) = w * b(C)$

Type 1: $b(K) = w_0 * b(C) + w_1$

Type k: $b(K) = w_1 + w_2 + w_3 + \cdots + w_k, k > 1$

Lemma 2.1. *Given a knot K such that K is a prime satellite knot of braid index 5, then the companion of K must be either a type 1 torus with braid index 2 or be a type 5 tours.*

Proof. Based on Birman and Menasco's [1] classification of Tori of either being of type 0, 1, or k we see that there are many tori to consider. We can elimanate cases where a braid index of 5 for a satelite knot is impossible using the equations listed above.

From these equations we get three cases.

Case 1: Type 0

Let A be a satellite knot of index 5 and companion torus T. For T to be a knotted torus of Type 0 then it must be of at least braid index 2 or greater, yet knot A's index is not a multiple of 2, 3, 4, or 6 or greater so, based on our equations, we can eliminate tori of these index. Therefore the only, knotted, type 0 torus which could contain a five braid would be a torus with index 5. Yet if the braid index of the knotted torus is five then the knot would be isotopic to a preferred longitude of the torus, and would no longer be satellite to it.

Therefore a type 0 torus could not be a companion to a 5 braid satellite.

Case 2: Type 1

For a Type 1 torus we can use our equation to determine what arrangements of weight and companion indices lead to a braid index of 5 for K.

w_0	w_1	$b(C)$
1	1	4
1	2	3
1	3	2
1	4	1
2	3	1
2	1	2
3	2	1
4	1	1

For the cases where $w_0 = 1$ we can observe that K is composite, therefore it is non-prime. For the cases where $b(C) = 1$ we can observe that the companion must be the unknot, being that only the unkot has a braid index of 1.

This leaves only the 2,1,2 arrangement remaining, which could take a prime arrangement since the wrapping number is 2 and the pattern could have a prime form as described in Theorem 1.1.

Case 3: Type k

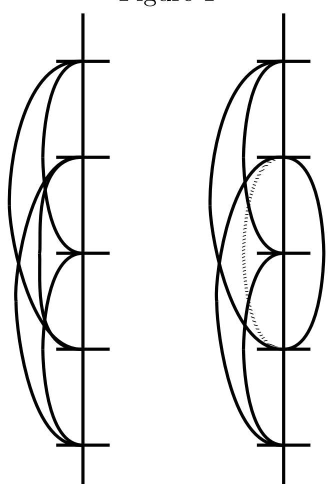
For a Type k torus we can see that we can have between 2 and 5 weights, where the number of weights is equal to the type number. We know that K has one component the knot will have a consistent weight across the torus. Therefore $b(K) = w_1 + w_2 + w_3 + \cdots + w_n = n * w_n$, since the braid index of our satellite is 5 then n and w_n must be multiples of 5 (1 or 5). If $w_n = 5$ then $n = 1$, yet we know that $n = k > 1$ so this is not possible. Therefore $w_n = 1$ and $n = 5$ which implies that $k = 5$. So a 5 braid satellite can only have a type k companion if $k = 5$. The pattern of the satellite is allowed to be either prime or the unknot, and the wrapping number is 2 so based on Theorem 1.1 this case can contain prime satellite knots. $\square$

The first step for describing a 5 braid satellite with companion type 5 torus is to determine what knots allow for a type 5 torus, which we can know is related to which knots have an arc index of 5.

Lemma 2.2. *The only Knot with an arc index of 5 is the Trefoil.*

Proof. Let K be a knot of arc index of 5, we can see from Cromwell's text [2] that the arcs of the diagram of K must have non-adjacent end points, and since we are working in $\mathbb{S}^3$ the top and bottom points are considered adjacent. Also only two arcs can meet at any point. From this we can see that there is only one way to connect the points on the binding axis to produce a knot. Now the projection of this diagram onto $\mathbb{R}^2$ with the greatest number of crossings is the diagram were all arcs lay on the same half-plane defined by the projection of the binding axis. For our diagram the maximum number of crossings is 5.

Figure 1



We know that each arc lays in a unique H_θ so therefore at least one arc can be moved from one half plane to the other and be a projection of an isotropic knot. Since this move reduces the number of crossings to 3 then a knot represented a 5 arc diagram can at most have three crossings. We know that the Trefoil has a 5 arc diagram and that it is the only non-trivial knot with three or less crossings, therefore it is the only knot with an arc index of 5.

□

Now we can use this information to determine a braid word for a 5 braid satellite with type 5 companion.

Theorem 2.1. *Let K be a 5 braid prime satellite with a type 5 companion, then K has a braid word of the form*

$$\mathbb{K}_5 = \{(\sigma_1\sigma_2\sigma_3^P\sigma_2^{-1}\sigma_1^{-1})(\sigma_2\sigma_3\sigma_4^Q\sigma_3^{-1}\sigma_2^{-1})(\sigma_1\sigma_2^R\sigma_1^{-1})(\sigma_2\sigma_3^S\sigma_2^{-1})(\sigma_3\sigma_4^T\sigma_3^{-1}) | \forall q \in \{P,Q,R,S,T\}; |q| > 0, \exists! p \in \{P,Q,R,S,T\}; p = 2h, h \in \mathbb{Z}\}$$

Where $\mathbb{K}_5$ is the set of these knots.

Proof Given a knot K such that K is a satellite with braid index 5 and companion of type 5 then K's companion is a trefoil by lemma 2.2. Therefore K is isotopic to Figure 2. This diagram can be projected down to $\mathbb{R}^2$ to produce a knot diagram of K, as shown in Figure 3.

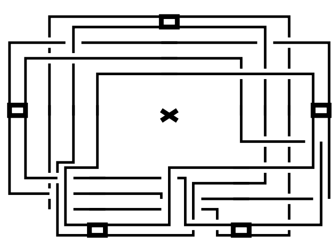


Figure 3

The knot K surrounds a point P that is the projection of the braid axis. If you cut the knot at points that intersect a given ray that begins at P then you will get the braid with braid word

$$\sigma_4^P\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2\sigma_4\sigma_3\sigma_4^Q\sigma_3^{-1}\sigma_2^{-1}\sigma_4^{-1}\sigma_3\sigma_4^R\sigma_1\sigma_3^{-1}\sigma_4\sigma_2\sigma_3\sigma_4^S\sigma_1\sigma_2\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_4\sigma_3^{-1}\sigma_4^{-1}\sigma_4^T\sigma_4\sigma_2\sigma_3\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1\sigma_2\sigma_3$$

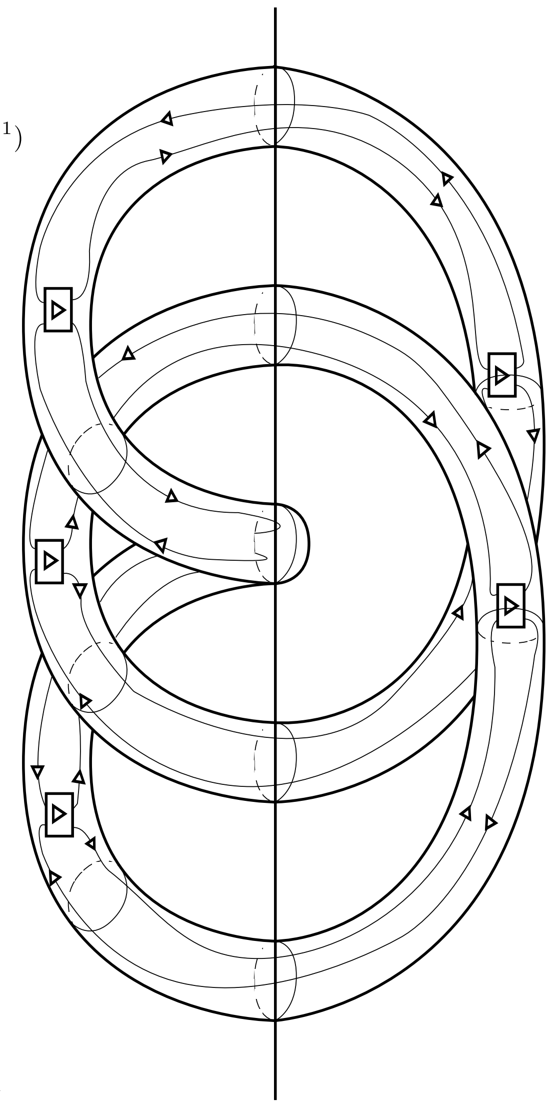


Figure 2

By simplifing the initial braid word algebraically, as demonstrated in Appendix 1, we are left with the following braid word.

$$(\sigma_1\sigma_2\sigma_3^P\sigma_2^{-1}\sigma_1^{-1})(\sigma_2\sigma_3\sigma_4^Q\sigma_3^{-1}\sigma_2^{-1})(\sigma_1\sigma_2^R\sigma_1^{-1})(\sigma_2\sigma_3^S\sigma_2^{-1})(\sigma_3\sigma_4^T\sigma_3^{-1})$$

If we look back to Figure 2 we can see that there are three possible arrangments for each braid box depending on whether there are an even or odd number of crossings or 0 crossings. The possible arrangments are depicted in Figure 4.

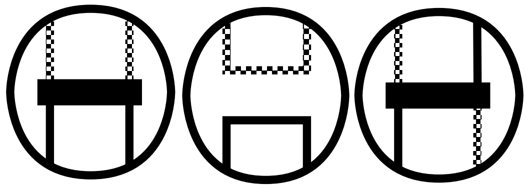


Figure 4: Even, 0 and Odd crossing numbers

If any braid box has 0 crossings then the knot misses a meridian disk in the interior of the torus and is no longer satellite to it. With more than one even box the braid becomes a multi component link and with all odd crossing numbers then the braid will also become a link. This then implies that the braid must have a sinlge box with an even number of crossings and all others must then be odd.

Therefore the structure of a satellite K with braid index 5 and companion of type 5 is
$\mathbb{K}_5 = \{(\sigma_1\sigma_2\sigma_3^P\sigma_2^{-1}\sigma_1^{-1})(\sigma_2\sigma_3\sigma_4^Q\sigma_3^{-1}\sigma_2^{-1})(\sigma_1\sigma_2^R\sigma_1^{-1})(\sigma_2\sigma_3^S\sigma_2^{-1})(\sigma_3\sigma_4^T\sigma_3^{-1})$
$|\forall q \in \{P,Q,R,S,T\}; |q| > 0, \exists! p \in \{P,Q,R,S,T\}; p = 2h, h \in \mathbb{Z}\}$.
Where $\mathbb{K}_5$ is the set of knots of this type.

□

We can see from Theorem 2.1 that the braid has a strong structure, but requires a long braid word to represent it, to make this form more concise we will now employ a shorthand that better illistrates the structure of this braid, we will call this shorthand a compressed sigma notation of the braid.

$$\sigma_{i,j}^X = (\sigma_i\sigma_{i+1}\dots\sigma_{j-2}\sigma_{j-1}^X\sigma_{j-2}^{-1}\dots\sigma_{i+1}^{-1}\sigma_i^{-1})$$

With this notation the braid word for Satellites with index 5 and companion of type 5 will be $(\sigma_{1,4}^P\sigma_{2,5}^Q\sigma_{1,3}^R\sigma_{2,4}^S\sigma_{3,5}^T)$, with the same bounds as previously stated.

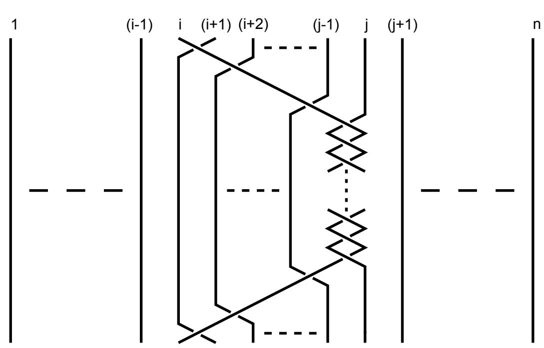


Figure 5: $\sigma_{i,j}$

Now if we combine our results from Theorem 2.1 and Lemma 2.1 we get the following result on the classification of 5 braid satellites,

Theorem 2.2. *Given a knot K with braid index 5 then the knot will be a member of the set will have one of the following braid words:*

$\beta_{3,5}(\sigma_3^m)(\sigma_2\sigma_3\sigma_1\sigma_2)^n$, *$n \geq 3$, $\beta_{3,5}$ being a braid box across the 3rd to 5th strands and the closure of $\beta_{3,5}$ is a non-trivial knot.*

$(\sigma_{1,4}^P\sigma_{2,5}^Q\sigma_{1,3}^R\sigma_{2,4}^S\sigma_{3,5}^T)$, *bounded as described in Theorem 2.1.*

Proof. From Lemma 2.1 we know that the only possible prime satellite knot K with a type 1 companion has a 2,1,2 configuration—so we know that the braid must include 2 winding strands through a 2 stranded torus, and 1 strand that cannot be removed by surgery, therefore the two strands wind through the non-trivial 2 braid of the torus as $(\sigma_2\sigma_3\sigma_1\sigma_2)^n$ where $n \geq$, or the torus is trivial or a link. The winding strands can cross themselves, σ_3^m and the non-winding strand must be included in the knot in a way that disallows surgery, so $\hat{\beta}_{3,5}$ must be a non-trivial knot with index 3. If K has a type 5 companion then it would have the form as shown in Theorem 2.1. □

From this we have a complete classification of prime satellite knots of index 5. We can see that the method used to create the knot with a type 5 companion can be expanded to higher cases, we will now use our methods from Theorem 2.1 to produce the braids of satellites with index 6 and comapanion of type 6. We know from Jin and Park [4] that the figure-eight has an arc index of 6, so to begin we will use it as our torus.

Theorem 2.3. *Let K be a satellite knot with companion figure-eight torus T of type 6, then K is in the set $\mathbb{K}_{6_1}$. Where*

$$\begin{aligned}\mathbb{K}_6 \quad = \quad & \{\sigma_{1,3}^A\sigma_{2,6}^B\sigma_{1,4}^C\sigma_{3,5}^D\sigma_{4,6}^E\sigma_{2,5}^F \mid \forall q \in \{A,B,C,D,E,F\} \\ & ;|q| > 0, \exists! p \in \{A,B,C,D,E,F\}; p = 2h, h \in \mathbb{Z}\}\end{aligned}$$

Proof. From Lemma 2.3 we know that there are two type 6 companion trefoils and from Jin and Park [4] we know that there is also a type six companion figure-eight. The diagrams above show the satellites contained in each possible torus, with a counter-clockwise orientation. So applying the same method as used for Theorem 2.1 we can produce the braid words above, the algebraic simplification, along with the projections of these knots onto $\mathbb{R}^2$ can be found in appendix 2. Similarly we can see that the braids require the same bounds as found in Theorem 2.1 through the same reasoning.

□

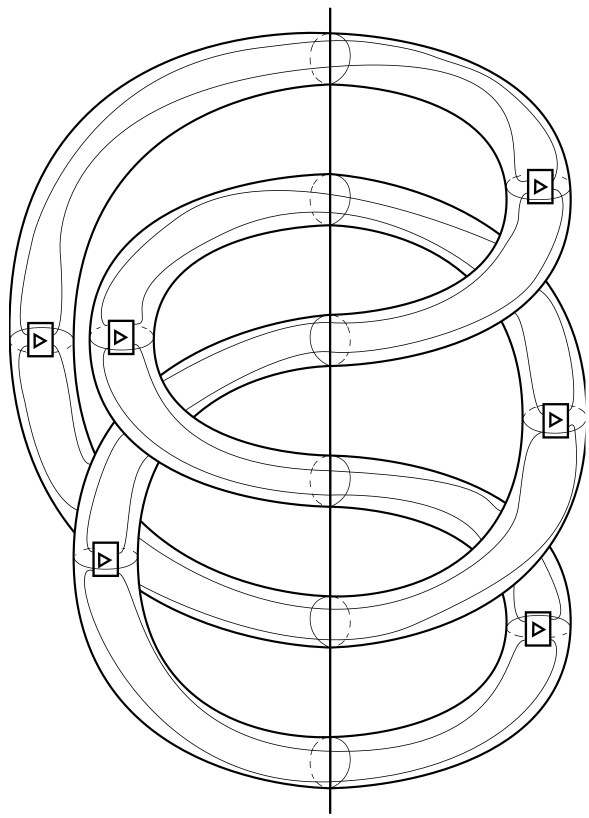
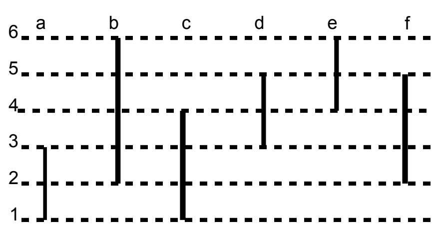
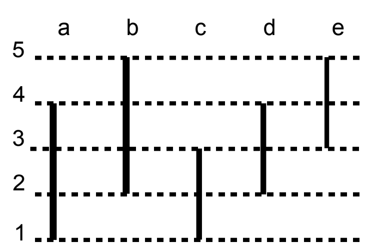


Figure 6: Satellite Contained Within a Type 6 Figure Eight

Looking at our results thus far we can begin to see some of the overarching structure to these sets of knots. First we can observe that there seems to be a definite relationship between the grid presentation, or grid diagram, and the braid structure. Second we can observe that the bounds on the numbers of crossings apear to be consistent for every braid of this type.





$(\sigma_{1,3}^{A}\sigma_{2,6}^{B}\sigma_{1,4}^{C}\sigma_{3,5}^{D}\sigma_{4,6}^{E}\sigma_{2,5}^{F})$ $(\sigma_{1,4}^{P}\sigma_{2,5}^{Q}\sigma_{1,3}^{R}\sigma_{2,4}^{S}\sigma_{3,5}^{T})$

Figure 7: Labelled Grid Presentation of Companions and the Corresponding Braids of the Satellites

Theorem 2.4. *Given a labelled grid presentation of a companion torus with labelled arcs* $\{a, b, c, \ldots, \omega\}$ *such that arc a spans* (X_a, Y_a)*, arc b spans* (X_b, Y_b) *and so on, where* X_* *and* Y_* *are the interger labels of the rows that the segment * begins and ends on,* $(X_* < Y_*)$*, then the braid word for the corresponding satellite would be*

$$(\sigma^{\alpha}_{X_a,Y_a}\sigma^{\beta}_{X_b,Y_b}\ldots\sigma^{\gamma}_{X_\omega,Y_\omega})$$

Proof Let K be an index k satellite knot such that its companion torus T is type k about the braid axis. Let H be a cylinder in the neighborhood of the braid axis.

If we project T onto the surface of H and then cut H along a vertical segment that does not contain the projection of T the result will be a grid presentation of T. Similarly If you project K onto the surface of H and cut H along a vertical segment that only contains horizontal segments of the projection of K, then the resulting diagram would be a valid braid diagram of K.

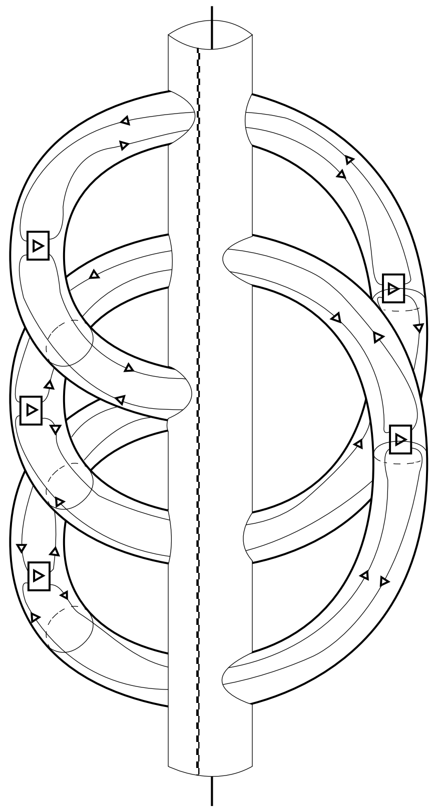


Figure 8: Satellite With Type 5 Trefoil, and the Cylinder H

The segments of K that are interior of the cylinder H would lay below the segments of K that lay exterior to H and the segments that lay above would be bounded in by the projection of T, therefore the resulting grid presentation and braid diagram would be related, where the arcs of T bound a braid box that crosses the strands that are positioned at either end of T. So the braid would be isotopic to the above braid word.

□

With this information we can now determine the bounds on the crossing numbers of satellites with index k and type k companion.

Theorem 2.5. *Given a satellite with braid index k and companion torus of type k then the structure of the braid will be as follows:*

$$\begin{aligned}\mathbb{K}_k \quad = \quad & \{\sigma^{\alpha}_{i,j}\sigma^{\beta}_{i,j}\sigma^{\gamma}_{i,j}\ldots\sigma^{\omega}_{i,j} | \forall q \in \{\alpha, \beta, \gamma, \ldots, \omega\}; \\ & |q| > 0 : \exists! p \in \{\alpha, \beta, \gamma, \ldots, \omega\}; p = 2h, h \in \mathbb{Z}\}\end{aligned}$$

where $\{\alpha, \beta, \gamma, \ldots, \omega\}$ *contains* k *elements and* i *and* j *represent unspecified strand numbers such that* $i < j$.

Proof. Let K be a satellite knot with braid index k and comapnion Torus T of type k around the braid axis. From Theorem 2.3 we can conclude that the braid word will have the form $(\sigma_{i,j}^{\alpha}\sigma_{i,j}^{\beta}\sigma_{i,j}^{\gamma}\ldots\sigma_{i,j}^{\omega})$ where i and j represent unkwown strand designations, $i < j$, and $\{\alpha, \beta, \gamma, \ldots, \omega\}$ contains k elements. We know that the local diagrams for each braid box can only have an even, odd, or zero crossing configuration as dipicted in Figure 4. If every box had an odd number of crossings then the strands would not reverse direction at any point within the torus, yet it is known that the link ineterior of the torus is bidirectional, which implies that K would be a two component link when every braid box has an odd number of crossings, therefore at least one must be even. If 2 or greater braid boxes had an even number of crossings then K would also form a link with more than one component. Finally if any braid box had zero crossings then K would not intersect every meridian within the interior of the torus and would no longer be satellite. Therefore $\forall q \in \{\alpha, \beta, \gamma, \ldots, \omega\}; |q| > 0 : \exists! p \in \{\alpha, \beta, \gamma, \ldots, \omega\}; p = 2h, h \in \mathbb{Z}$

□

Now we have a complete method for constructing satellite knots with index k and companion torus of type k which we can apply to create more braid words. Such as the case where $k = 7$.

Result 2.1. *Using our results and results from the work of Jin and Park [4] we can determine the structure of any satellite knot of braid index 7 and companion being of type 7 and arc index 7 to be a subset one of the following sets:*

Knot 5_1**:** $\beta_{7_1} = \{\sigma_{27}^{P}\sigma_{16}^{Q}\sigma_{57}^{R}\sigma_{46}^{S}\sigma_{35}^{T}\sigma_{24}^{U}\sigma_{13}^{V}\}$

Knot 5_2**:** $\beta_{7_2} = \{\sigma_{16}^{P}\sigma_{57}^{Q}\sigma_{46}^{R}\sigma_{25}^{S}\sigma_{13}^{T}\sigma_{24}^{U}\sigma_{37}^{V}\}$

Knot 8_{19}**:** $\beta_{7_3} = \{\sigma_{37}^{P}\sigma_{26}^{Q}\sigma_{15}^{R}\sigma_{47}^{S}\sigma_{36}^{T}\sigma_{25}^{U}\sigma_{14}^{V}\}$

These braids have the same bounds as shown in Theorem 2.4 and have a type 7 companion of the knot notted to its left.

3 Conclusion

In conclusion our study of this class of satellite knots has produced a complete classification of prime five braids and a methodology through which satellites can be constructed from any given type k torus, these satellites have a braid index of k and are bidirectional within the torus. These knots have fairly interesting properties and provide an insight into satellite knots as a whole.

3.1 Open Questions

Continuing this study I plan to investigate several open questions on the primality of these knots and multi-component links of similar structure, some of these open questions include:

1. Are all satellite knots of this structure prime?
2. What does the pattern of these knots (pretzel) tell us about their structure and what is the relationship between this pattern and the type k torus?
3. What properties would a satellite link with companion torus of type k have?
4. What properties would a satellite knot of index $2k$ and companion of type k have?
5. Can satellites with companion of type k, but arc index $k-1$ have a braid index of $a(k-1)$?
6. Is the closed braid with the structure

$$\begin{aligned}\mathbb{K}_k &= \{\sigma^{\alpha}_{i_1,j_1}\sigma^{\beta}_{i_2,j_2}\sigma^{\gamma}_{i_3,j_3}\ldots\sigma^{\omega}_{i_n,j_n}|\forall q \in \{\alpha,\beta,\gamma,\ldots,\omega\};\\ &\quad |q| > 0 : \forall p \in \{\alpha,\beta,\gamma,\ldots,\omega\}; p \neq 2h, h \in \mathbb{Z}\}\end{aligned}$$

   a non trivial link?
7. Can you determine the companion torus of a given satellite knot?
8. Can you determine the companion torus for a given satellite link?

3.2 Acknowledgements

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4 Appendix

For this appendix the symbol $\cong$ will be used to represent that the two braid words are isotopic when closed.

4.1 Theorem 2.1: Algebraic Simplification of Braid

$$\begin{aligned}\mathbb{K}_5 &= \sigma_4^P\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2\sigma_4\sigma_3\sigma_4^Q\sigma_3^{-1}\sigma_2^{-1}\sigma_4^{-1}\sigma_3\sigma_4^R\sigma_1\sigma_3^{-1}\sigma_4\sigma_2\sigma_3\\ &\quad\sigma_4^S\sigma_1\sigma_2\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_4\sigma_3^{-1}\sigma_4^{-1}\sigma_4^T\sigma_4\sigma_2\sigma_3\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1\sigma_2\sigma_3\end{aligned}$$

$$\sigma_1\sigma_3^{-1}\sigma_4\sigma_2\sigma_3\sigma_4^S\sigma_1 = \sigma_3^{-1}\sigma_4\sigma_1\sigma_2\sigma_1\sigma_3\sigma_4^S$$

$$\begin{aligned}&= \sigma_4^P\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2\sigma_4\sigma_3\sigma_4^Q\sigma_3^{-1}\sigma_2^{-1}\sigma_4^{-1}\sigma_3\sigma_4^R\sigma_3^{-1}\sigma_4\sigma_1\sigma_2\sigma_1\sigma_3\\ &\quad\sigma_4^S\sigma_2\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_4\sigma_3^{-1}\sigma_4^{-1}\sigma_4^T\sigma_4\sigma_2\sigma_3\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1\sigma_2\sigma_3\end{aligned}$$

$$\sigma_1\sigma_2\sigma_1\sigma_3\sigma_4^S\sigma_2 = \sigma_2\sigma_1\sigma_2\sigma_3\sigma_2\sigma_4^S$$

$$\begin{aligned}&= \sigma_4^P\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2\sigma_4\sigma_3\sigma_4^Q\sigma_3^{-1}\sigma_2^{-1}\sigma_4^{-1}\sigma_3\sigma_4^R\sigma_3^{-1}\sigma_4\sigma_2\sigma_1\sigma_2\sigma_3\sigma_2\\ &\quad\sigma_4^S\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_4\sigma_3^{-1}\sigma_4^{-1}\sigma_4^T\sigma_4\sigma_2\sigma_3\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1\sigma_2\sigma_3\end{aligned}$$

$$\sigma_1\sigma_2\sigma_3\sigma_2\sigma_4^S = \sigma_3\sigma_1\sigma_2\sigma_3\sigma_4^S$$

$$\begin{aligned}&= \sigma_4^P\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2\sigma_4\sigma_3\sigma_4^Q\sigma_3^{-1}\sigma_2^{-1}\sigma_4^{-1}\sigma_3\sigma_4^R\sigma_3^{-1}\sigma_4\sigma_2\sigma_3\sigma_1\sigma_2\sigma_3\\ &\quad\sigma_4^S\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_4\sigma_3^{-1}\sigma_4^{-1}\sigma_4^T\sigma_4\sigma_2\sigma_3\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1\sigma_2\sigma_3\end{aligned}$$

$$\sigma_4^{-1}\sigma_4^T\sigma_4\sigma_2\sigma_3\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1\sigma_2\sigma_3 = \sigma_4^T\sigma_2\sigma_3\sigma_4^{-1}\sigma_3^{-1}\sigma_1\sigma_2\sigma_3\sigma_1^{-1}$$

$$\begin{aligned}&= \sigma_4^P\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2\sigma_4\sigma_3\sigma_4^Q\sigma_3^{-1}\sigma_2^{-1}\sigma_4^{-1}\sigma_3\sigma_4^R\sigma_3^{-1}\sigma_4\sigma_2\sigma_3\sigma_1\sigma_2\sigma_3\\ &\quad\sigma_4^S\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_4\sigma_3^{-1}\sigma_4^T\sigma_2\sigma_3\sigma_4^{-1}\sigma_3^{-1}\sigma_1\sigma_2\sigma_3\sigma_1^{-1}\end{aligned}$$

$$\sigma_1^{-1}\sigma_4\sigma_3^{-1}\sigma_4^T\sigma_2\sigma_3\sigma_4^{-1}\sigma_3^{-1}\sigma_1\sigma_2\sigma_3\sigma_1^{-1} = \sigma_4\sigma_3^{-1}\sigma_4^T\sigma_1^{-1}\sigma_2\sigma_1\sigma_3\sigma_2\sigma_4^{-1}\sigma_3\sigma_2^{-1}\sigma_1^{-1}$$

$$\begin{aligned}&= \sigma_4^P\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2\sigma_4\sigma_3\sigma_4^Q\sigma_3^{-1}\sigma_2^{-1}\sigma_4^{-1}\sigma_3\sigma_4^R\sigma_3^{-1}\sigma_4\sigma_2\sigma_3\sigma_1\sigma_2\sigma_3\\ &\quad\sigma_4^S\sigma_3^{-1}\sigma_2^{-1}\sigma_4\sigma_3^{-1}\sigma_4^T\sigma_1^{-1}\sigma_2\sigma_1\sigma_3\sigma_2\sigma_4^{-1}\sigma_3\sigma_2^{-1}\sigma_1^{-1}\end{aligned}$$

$$\sigma_2^{-1}\sigma_4\sigma_3^{-1}\sigma_4^T\sigma_1^{-1}\sigma_2\sigma_1\sigma_3\sigma_2\sigma_4^{-1} = \sigma_4\sigma_2^{-1}\sigma_3^{-1}\sigma_2\sigma_4^T\sigma_1\sigma_3\sigma_2\sigma_3^{-1}\sigma_4^{-1}$$

$$\begin{aligned}&= \sigma_4^P\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2\sigma_4\sigma_3\sigma_4^Q\sigma_3^{-1}\sigma_2^{-1}\sigma_4^{-1}\sigma_3\sigma_4^R\sigma_3^{-1}\sigma_4\sigma_2\sigma_3\sigma_1\sigma_2\sigma_3\\ &\quad\sigma_4^S\sigma_3^{-1}\sigma_4\sigma_2^{-1}\sigma_3^{-1}\sigma_2\sigma_4^T\sigma_1\sigma_3\sigma_2\sigma_3^{-1}\sigma_4^{-1}\sigma_3\sigma_2^{-1}\sigma_1^{-1}\end{aligned}$$

$$\sigma_2^{-1}\sigma_3^{-1}\sigma_2\sigma_4^T\sigma_1\sigma_3\sigma_2\sigma_3^{-1}\sigma_4^{-1}\sigma_3 = \sigma_3\sigma_2^{-1}\sigma_3^{-1}\sigma_4^T\sigma_1\sigma_3\sigma_2\sigma_4\sigma_3^{-1}\sigma_4^{-1}$$

$$\begin{aligned}&= \sigma_4^P\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2\sigma_4\sigma_3\sigma_4^Q\sigma_3^{-1}\sigma_2^{-1}\sigma_4^{-1}\sigma_3\sigma_4^R\sigma_3^{-1}\sigma_4\sigma_2\sigma_3\sigma_1\sigma_2\sigma_3\\ &\quad\sigma_4^S\sigma_3^{-1}\sigma_4\sigma_3\sigma_2^{-1}\sigma_3^{-1}\sigma_4^T\sigma_1\sigma_3\sigma_2\sigma_4\sigma_3^{-1}\sigma_4^{-1}\sigma_2^{-1}\sigma_1^{-1}\end{aligned}$$

$$\sigma_4^S\sigma_3^{-1}\sigma_4\sigma_3\sigma_2^{-1} = \sigma_4^{S+1}\sigma_3\sigma_2^{-1}\sigma_4^{-1}$$

$$\begin{aligned}&= \sigma_4^P\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2\sigma_4\sigma_3\sigma_4^Q\sigma_3^{-1}\sigma_2^{-1}\sigma_4^{-1}\sigma_3\sigma_4^R\sigma_3^{-1}\sigma_4\sigma_2\sigma_3\sigma_1\sigma_2\sigma_3\\ &\quad\sigma_4^{S+1}\sigma_3\sigma_2^{-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_4^T\sigma_1\sigma_3\sigma_2\sigma_4\sigma_3^{-1}\sigma_4^{-1}\sigma_2^{-1}\sigma_1^{-1}\end{aligned}$$

$$\sigma_4^{-1}\sigma_3^{-1}\sigma_4^T = \sigma_3^T\sigma_4^{-1}\sigma_3^{-1}$$

$$= \sigma_4^P \sigma_3^{-1} \sigma_2^{-1} \sigma_1^{-1} \sigma_4^{-1} \sigma_3^{-1} \sigma_2 \sigma_4 \sigma_3 \sigma_4^Q \sigma_3^{-1} \sigma_2^{-1} \sigma_4^{-1} \sigma_3 \sigma_4^R \sigma_3^{-1} \sigma_4 \sigma_2 \sigma_3 \sigma_1 \sigma_2 \sigma_3$$
$$\sigma_4^{S+1} \sigma_3 \sigma_2^{-1} \sigma_3^T \sigma_4^{-1} \sigma_3^{-1} \sigma_1 \sigma_3 \sigma_2 \sigma_4 \sigma_3^{-1} \sigma_4^{-1} \sigma_2^{-1} \sigma_1^{-1}$$

$$\sigma_4^{-1} \sigma_3^{-1} \sigma_1 \sigma_3 \sigma_2 \sigma_4 = \sigma_1 \sigma_2$$

$$= \sigma_4^P \sigma_3^{-1} \sigma_2^{-1} \sigma_1^{-1} \sigma_4^{-1} \sigma_3^{-1} \sigma_2 \sigma_4 \sigma_3 \sigma_4^Q \sigma_3^{-1} \sigma_2^{-1} \sigma_4^{-1} \sigma_3 \sigma_4^R \sigma_3^{-1} \sigma_4 \sigma_2 \sigma_3 \sigma_1 \sigma_2 \sigma_3$$
$$\sigma_4^{S+1} \sigma_3 \sigma_2^{-1} \sigma_3^T \sigma_1 \sigma_2 \sigma_3^{-1} \sigma_4^{-1} \sigma_2^{-1} \sigma_1^{-1}$$

The closed braid $\alpha\beta\gamma$ is isotopic to the closed braid $\gamma\alpha\beta$.
$\sigma_4^P \ldots \sigma_1 \sigma_2 \sigma_3^{-1} \sigma_4^{-1} \sigma_2^{-1} \sigma_1^{-1} \cong \sigma_4^{P-1} \ldots \sigma_1 \sigma_3^{-1} \sigma_2^{-1} \sigma_3 \sigma_1^{-1}$

$$\cong \sigma_4^{P-1} \sigma_3^{-1} \sigma_2^{-1} \sigma_1^{-1} \sigma_4^{-1} \sigma_3^{-1} \sigma_2 \sigma_4 \sigma_3 \sigma_4^Q \sigma_3^{-1} \sigma_2^{-1} \sigma_4^{-1} \sigma_3 \sigma_4^R \sigma_3^{-1} \sigma_4 \sigma_2 \sigma_3 \sigma_1 \sigma_2 \sigma_3$$
$$\sigma_4^{S+1} \sigma_3 \sigma_2^{-1} \sigma_3^T \sigma_1 \sigma_3^{-1} \sigma_2^{-1} \sigma_3 \sigma_1^{-1}$$

$$\sigma_3^T \sigma_1 \sigma_3^{-1} = \sigma_3^{T-1} \sigma_1$$

$$= \sigma_4^{P-1} \sigma_3^{-1} \sigma_2^{-1} \sigma_1^{-1} \sigma_4^{-1} \sigma_3^{-1} \sigma_2 \sigma_4 \sigma_3 \sigma_4^Q \sigma_3^{-1} \sigma_2^{-1} \sigma_4^{-1} \sigma_3 \sigma_4^R \sigma_3^{-1} \sigma_4 \sigma_2 \sigma_3 \sigma_1 \sigma_2 \sigma_3$$
$$\sigma_4^{S+1} \sigma_3 \sigma_2^{-1} \sigma_3^{T-1} \sigma_1 \sigma_2^{-1} \sigma_3 \sigma_1^{-1}$$

$$\sigma_4 \sigma_2 \sigma_3 \sigma_1 = \sigma_2 \sigma_4 \sigma_1 \sigma_3$$

$$= \sigma_4^{P-1} \sigma_3^{-1} \sigma_2^{-1} \sigma_1^{-1} \sigma_4^{-1} \sigma_3^{-1} \sigma_2 \sigma_4 \sigma_3 \sigma_4^Q \sigma_3^{-1} \sigma_2^{-1} \sigma_4^{-1} \sigma_3 \sigma_4^R \sigma_3^{-1} \sigma_2 \sigma_4 \sigma_1 \sigma_3 \sigma_2 \sigma_3$$
$$\sigma_4^{S+1} \sigma_3 \sigma_2^{-1} \sigma_3^{T-1} \sigma_1 \sigma_2^{-1} \sigma_3 \sigma_1^{-1}$$

$$\sigma_4 \sigma_1 \sigma_3 \sigma_2 \sigma_3 \sigma_4^{S+1} \sigma_3 = \sigma_1 \sigma_2 \sigma_4 \sigma_3 \sigma_4^{S+1} \sigma_2 \sigma_3$$

$$= \sigma_4^{P-1} \sigma_3^{-1} \sigma_2^{-1} \sigma_1^{-1} \sigma_4^{-1} \sigma_3^{-1} \sigma_2 \sigma_4 \sigma_3 \sigma_4^Q \sigma_3^{-1} \sigma_2^{-1} \sigma_4^{-1} \sigma_3 \sigma_4^R \sigma_3^{-1} \sigma_2 \sigma_1 \sigma_2 \sigma_4 \sigma_3$$
$$\sigma_4^{S+1} \sigma_2 \sigma_3 \sigma_2^{-1} \sigma_3^{T-1} \sigma_1 \sigma_2^{-1} \sigma_3 \sigma_1^{-1}$$

$$\sigma_2 \sigma_1 \sigma_2 \sigma_4 \sigma_3 \sigma_4^{S+1} \sigma_2 \sigma_3 \sigma_2^{-1} \sigma_3^{T-1} = \sigma_1 \sigma_2 \sigma_1 \sigma_3^{S+1} \sigma_4 \sigma_3 \sigma_3^{-1} \sigma_2 \sigma_3^T$$

$$= \sigma_4^{P-1} \sigma_3^{-1} \sigma_2^{-1} \sigma_1^{-1} \sigma_4^{-1} \sigma_3^{-1} \sigma_2 \sigma_4 \sigma_3 \sigma_4^Q \sigma_3^{-1} \sigma_2^{-1} \sigma_4^{-1} \sigma_3 \sigma_4^R \sigma_3^{-1} \sigma_1 \sigma_2 \sigma_1 \sigma_3^{S+1}$$
$$\sigma_4 \sigma_3 \sigma_3^{-1} \sigma_2 \sigma_3^T \sigma_1 \sigma_2^{-1} \sigma_3 \sigma_1^{-1}$$

$$\sigma_1 \sigma_3^{S+1} \sigma_4 \sigma_3 \sigma_3^{-1} \sigma_2 \sigma_3^T \sigma_1 \sigma_2^{-1} \sigma_3 \sigma_1^{-1} = \sigma_3^{S+1} \sigma_4 \sigma_1 \sigma_2 \sigma_3^T \sigma_2^{-1} \sigma_1^{-1} \sigma_2 \sigma_3$$

$$= \sigma_4^{P-1} \sigma_3^{-1} \sigma_2^{-1} \sigma_1^{-1} \sigma_4^{-1} \sigma_3^{-1} \sigma_2 \sigma_4 \sigma_3 \sigma_4^Q \sigma_3^{-1} \sigma_2^{-1} \sigma_4^{-1} \sigma_3 \sigma_4^R \sigma_3^{-1} \sigma_1 \sigma_2 \sigma_3^{S+1}$$
$$\sigma_4 \sigma_1 \sigma_2 \sigma_3^T \sigma_2^{-1} \sigma_1^{-1} \sigma_2 \sigma_3$$

$$\sigma_2 \sigma_4 \sigma_3 \sigma_4^Q \sigma_3^{-1} = \sigma_2 \sigma_3^Q \sigma_4 \sigma_3 \sigma_3^{-1}$$

$$= \sigma_4^{P-1} \sigma_3^{-1} \sigma_2^{-1} \sigma_1^{-1} \sigma_4^{-1} \sigma_3^{-1} \sigma_2 \sigma_3^Q \sigma_4 \sigma_3 \sigma_3^{-1} \sigma_2^{-1} \sigma_4^{-1} \sigma_3 \sigma_4^R \sigma_3^{-1} \sigma_1 \sigma_2 \sigma_3^{S+1}$$
$$\sigma_4 \sigma_1 \sigma_2 \sigma_3^T \sigma_2^{-1} \sigma_1^{-1} \sigma_2 \sigma_3$$

$$\sigma_4 \sigma_3 \sigma_3^{-1} \sigma_2^{-1} \sigma_4^{-1} = \sigma_2^{-1}$$

$$= \sigma_4^{P-1} \sigma_3^{-1} \sigma_2^{-1} \sigma_1^{-1} \sigma_4^{-1} \sigma_3^{-1} \sigma_2 \sigma_3^Q \sigma_2^{-1} \sigma_3 \sigma_4^R \sigma_3^{-1} \sigma_1 \sigma_2 \sigma_3^{S+1} \sigma_4 \sigma_1 \sigma_2 \sigma_3^T \sigma_2^{-1} \sigma_1^{-1} \sigma_2 \sigma_3$$

$$\sigma_3^{-1} \sigma_1 \sigma_2 \sigma_3^{S+1} = \sigma_1 \sigma_2 \sigma_3 \sigma_2^{-1} \sigma_3^S$$

$$= \sigma_4^{P-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2\sigma_3^{Q}\sigma_2^{-1}\sigma_3\sigma_4^{R}\sigma_1\sigma_2\sigma_3\sigma_2^{-1}\sigma_3^{S}\sigma_4\sigma_1\sigma_2\sigma_3^{T}\sigma_2^{-1}\sigma_1^{-1}\sigma_2\sigma_3$$

$$\sigma_3\sigma_4^{R}\sigma_1\sigma_2\sigma_3\sigma_2^{-1} = \sigma_1\sigma_3\sigma_2\sigma_4^{R}\sigma_3\sigma_4\sigma_2^{-1}\sigma_4^{-1}$$

$$= \sigma_4^{P-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2\sigma_3^{Q}\sigma_2^{-1}\sigma_1\sigma_3\sigma_2\sigma_4^{R}\sigma_3\sigma_4\sigma_2^{-1}\sigma_4^{-1}\sigma_3^{S}\sigma_4\sigma_1\sigma_2\sigma_3^{T}\sigma_2^{-1}\sigma_1^{-1}\sigma_2\sigma_3$$

$$\sigma_4^{R}\sigma_3\sigma_4\sigma_2^{-1} = \sigma_3\sigma_4\sigma_3^{R}\sigma_2^{-1}$$

$$= \sigma_4^{P-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2\sigma_3^{Q}\sigma_2^{-1}\sigma_1\sigma_3\sigma_2\sigma_3\sigma_4\sigma_3^{R}\sigma_2^{-1}\sigma_4^{-1}\sigma_3^{S}\sigma_4\sigma_1\sigma_2\sigma_3^{T}\sigma_2^{-1}\sigma_1^{-1}\sigma_2\sigma_3$$

$$\sigma_1^{-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2\sigma_3^{Q}\sigma_2^{-1}\sigma_1\sigma_3\sigma_2\sigma_3\sigma_4 = \sigma_4^{-1}\sigma_3^{-1}\sigma_1^{-1}\sigma_2\sigma_3^{Q}\sigma_2^{-1}\sigma_1\sigma_2\sigma_3\sigma_4\sigma_2$$

$$= \sigma_4^{P-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_1^{-1}\sigma_2\sigma_3^{Q}\sigma_2^{-1}\sigma_1\sigma_2\sigma_3\sigma_4\sigma_2\sigma_3^{R}\sigma_2^{-1}\sigma_4^{-1}\sigma_3^{S}\sigma_4\sigma_1\sigma_2\sigma_3^{T}\sigma_2^{-1}\sigma_1^{-1}\sigma_2\sigma_3$$

$$\sigma_1^{-1}\sigma_2\sigma_3^{Q}\sigma_2^{-1}\sigma_1\sigma_2 = \sigma_2\sigma_1\sigma_2^{-1}\sigma_3^{Q}\sigma_2\sigma_1^{-1}$$

$$= \sigma_4^{P-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2\sigma_1\sigma_2^{-1}\sigma_3^{Q}\sigma_2\sigma_1^{-1}\sigma_3\sigma_4\sigma_2\sigma_3^{R}\sigma_2^{-1}\sigma_4^{-1}\sigma_3^{S}\sigma_4\sigma_1\sigma_2\sigma_3^{T}\sigma_2^{-1}\sigma_1^{-1}\sigma_2\sigma_3$$

$$\sigma_2^{-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2 = \sigma_4^{-1}\sigma_3\sigma_2^{-1}\sigma_3^{-1}$$

$$= \sigma_4^{P-1}\sigma_3^{-1}\sigma_4^{-1}\sigma_3\sigma_2^{-1}\sigma_3^{-1}\sigma_1\sigma_2^{-1}\sigma_3^{Q}\sigma_2\sigma_1^{-1}\sigma_3\sigma_4\sigma_2\sigma_3^{R}\sigma_2^{-1}\sigma_4^{-1}\sigma_3^{S}\sigma_4\sigma_1\sigma_2\sigma_3^{T}\sigma_2^{-1}\sigma_1^{-1}\sigma_2\sigma_3$$

$$\sigma_3\sigma_2^{-1}\sigma_3^{-1}\sigma_1\sigma_2^{-1}\sigma_3^{Q}\sigma_2\sigma_1^{-1}\sigma_3 = \sigma_2^{-1}\sigma_3^{-1}\sigma_1\sigma_2^{-1}\sigma_3^{Q}\sigma_2\sigma_3\sigma_1^{-1}$$

$$= \sigma_4^{P}\sigma_3^{-1}\sigma_4^{-1}\sigma_2^{-1}\sigma_3^{-1}\sigma_1\sigma_2^{-1}\sigma_3^{Q}\sigma_2\sigma_3\sigma_1^{-1}\sigma_4\sigma_2\sigma_3^{R}\sigma_2^{-1}\sigma_4^{-1}\sigma_3^{S}\sigma_4\sigma_1\sigma_2\sigma_3^{T}\sigma_2^{-1}\sigma_1^{-1}\sigma_2\sigma_3$$

$$\sigma_2^{-1}\sigma_3^{Q}\sigma_2\sigma_3 = \sigma_3\sigma_2^{Q}$$

$$= \sigma_4^{P}\sigma_3^{-1}\sigma_4^{-1}\sigma_2^{-1}\sigma_3^{-1}\sigma_1\sigma_3\sigma_2^{Q}\sigma_1^{-1}\sigma_4\sigma_2\sigma_3^{R}\sigma_2^{-1}\sigma_4^{-1}\sigma_3^{S}\sigma_4\sigma_1\sigma_2\sigma_3^{T}\sigma_2^{-1}\sigma_1^{-1}\sigma_2\sigma_3$$

$$\sigma_4^{-1}\sigma_2^{-1}\sigma_3^{-1}\sigma_1\sigma_3\sigma_2^{Q}\sigma_1^{-1}\sigma_4 = \sigma_2^{-1}\sigma_1\sigma_2^{Q}\sigma_1^{-1}$$

$$= \sigma_4^{P}\sigma_3^{-1}\sigma_2^{-1}\sigma_1\sigma_2^{Q}\sigma_1^{-1}\sigma_2\sigma_3^{R}\sigma_2^{-1}\sigma_4^{-1}\sigma_3^{S}\sigma_4\sigma_1\sigma_2\sigma_3^{T}\sigma_2^{-1}\sigma_1^{-1}\sigma_2\sigma_3$$

$$\sigma_4^{-1}\sigma_3^{S}\sigma_4 = \sigma_3\sigma_3^{-1}\sigma_4^{-1}\sigma_3^{S}\sigma_4$$

$$= \sigma_4^{P}\sigma_3^{-1}\sigma_2^{-1}\sigma_1\sigma_2^{Q}\sigma_1^{-1}\sigma_2\sigma_3^{R}\sigma_2^{-1}\sigma_3\sigma_3^{-1}\sigma_4^{-1}\sigma_3^{S}\sigma_4\sigma_1\sigma_2\sigma_3^{T}\sigma_2^{-1}\sigma_1^{-1}\sigma_2\sigma_3$$

$$\sigma_3\sigma_3^{-1}\sigma_4^{-1}\sigma_3^{S}\sigma_4 = \sigma_3\sigma_4^{S}\sigma_3^{-1}\sigma_4^{-1}\sigma_4$$

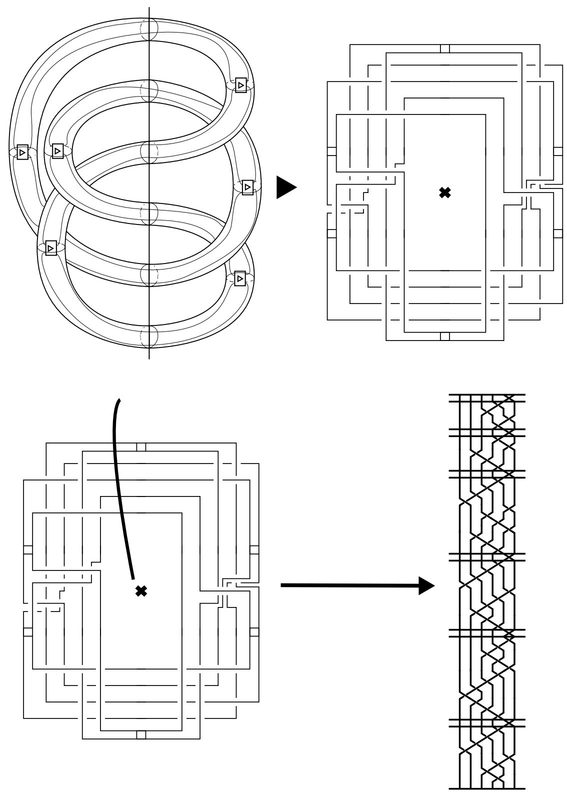
$$= \sigma_4^{P}\sigma_3^{-1}\sigma_2^{-1}\sigma_1\sigma_2^{Q}\sigma_1^{-1}\sigma_2\sigma_3^{R}\sigma_2^{-1}\sigma_3\sigma_4^{S}\sigma_3^{-1}\sigma_4^{-1}\sigma_4\sigma_1\sigma_2\sigma_3^{T}\sigma_2^{-1}\sigma_1^{-1}\sigma_2\sigma_3$$

$$\sigma_3\sigma_4^{S}\sigma_3^{-1}\sigma_4^{-1}\sigma_4 = \sigma_3\sigma_4^{S}\sigma_3^{-1}$$

$$\mathbb{K}_5 \cong (\sigma_2\sigma_3\sigma_4^{P}\sigma_3^{-1}\sigma_2^{-1})(\sigma_1\sigma_2^{Q}\sigma_1^{-1})(\sigma_2\sigma_3^{R}\sigma_2^{-1})(\sigma_3\sigma_4^{S}\sigma_3^{-1})(\sigma_1\sigma_2\sigma_3^{T}\sigma_2^{-1}\sigma_1^{-1})$$

4.2 Theorem 2.3: Algebraic Simpflication of Braids and Related Diagrams

4.2.1 Diagrams



4.2.2 Algebraic Simplification

$$\begin{aligned}\mathbb{K}_{6_1} = {}& \sigma_5^A\sigma_4^{-1}\sigma_3^{-1}\sigma_5^{-1}\sigma_4\sigma_5^B\sigma_4^{-1}\sigma_5\sigma_2\sigma_3\sigma_4\sigma_5^C\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1\sigma_5^{-1}\sigma_3\sigma_4\sigma_5\sigma_2\sigma_3\sigma_4\\ & \sigma_5^D\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_5^{-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2\sigma_3\sigma_4\sigma_5^E\sigma_5\sigma_4\sigma_3\sigma_4\sigma_5^{-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1\\ & \sigma_2\sigma_3\sigma_4\sigma_5^F\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_5^{-1}\sigma_4^{-1}\sigma_3\sigma_4\end{aligned}$$

$$\begin{aligned}\sigma_5^E\sigma_5\sigma_4\sigma_3\sigma_4\sigma_5^{-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1\sigma_2\sigma_3\sigma_4\sigma_5^F = {}\\ \sigma_5^{E+1}\sigma_4\sigma_3\sigma_4\sigma_1\sigma_2\sigma_3\sigma_4\sigma_5\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_5^{F-1}\end{aligned}$$

$$\begin{aligned}= {}& \sigma_5^A\sigma_4^{-1}\sigma_3^{-1}\sigma_5^{-1}\sigma_4\sigma_5^B\sigma_4^{-1}\sigma_5\sigma_2\sigma_3\sigma_4\sigma_5^C\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1\sigma_5^{-1}\sigma_3\sigma_4\sigma_5\sigma_2\sigma_3\sigma_4\\ & \sigma_5^D\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_5^{-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2\sigma_3\sigma_4\sigma_5^{E+1}\sigma_4\sigma_3\sigma_4\sigma_1\sigma_2\sigma_3\sigma_4\sigma_5\sigma_4^{-1}\sigma_3^{-1}\\ & \sigma_2^{-1}\sigma_1^{-1}\sigma_5^{F-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_5^{-1}\sigma_4^{-1}\sigma_3\sigma_4\end{aligned}$$

$$\sigma_4\sigma_3\sigma_4\sigma_1\sigma_2\sigma_3\sigma_4\sigma_5 = \sigma_1\sigma_2\sigma_3\sigma_4\sigma_5\sigma_3\sigma_2\sigma_3$$

$$
\begin{aligned}
&= \sigma_5^A\sigma_4^{-1}\sigma_3^{-1}\sigma_5^{-1}\sigma_4\sigma_5^B\sigma_4^{-1}\sigma_5\sigma_2\sigma_3\sigma_4\sigma_5^C\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1\sigma_5^{-1}\sigma_3\sigma_4\sigma_5\sigma_2\sigma_3\sigma_4 \\
&\quad \sigma_5^D\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_5^{-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2\sigma_3\sigma_4\sigma_5^{E+1}\sigma_1\sigma_2\sigma_3\sigma_4\sigma_5\sigma_3\sigma_2\sigma_3\sigma_4^{-1}\sigma_3^{-1} \\
&\quad \sigma_2^{-1}\sigma_1^{-1}\sigma_5^{F-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_5^{-1}\sigma_4^{-1}\sigma_3\sigma_4
\end{aligned}
$$

$$\sigma_2\sigma_3\sigma_4\sigma_5^{E+1}\sigma_1\sigma_2\sigma_3\sigma_4\sigma_5 = \sigma_1\sigma_2\sigma_3\sigma_4\sigma_5\sigma_1\sigma_2\sigma_3\sigma_4^{E+1}$$

$$
\begin{aligned}
&= \sigma_5^A\sigma_4^{-1}\sigma_3^{-1}\sigma_5^{-1}\sigma_4\sigma_5^B\sigma_4^{-1}\sigma_5\sigma_2\sigma_3\sigma_4\sigma_5^C\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1\sigma_5^{-1}\sigma_3\sigma_4\sigma_5\sigma_2\sigma_3\sigma_4 \\
&\quad \sigma_5^D\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_5^{-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_1\sigma_2\sigma_3\sigma_4\sigma_5\sigma_1\sigma_2\sigma_3\sigma_4^{E+1}\sigma_3\sigma_2\sigma_3\sigma_4^{-1}\sigma_3^{-1} \\
&\quad \sigma_2^{-1}\sigma_1^{-1}\sigma_5^{F-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_5^{-1}\sigma_4^{-1}\sigma_3\sigma_4
\end{aligned}
$$

$$\sigma_5^D\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_5^{-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_1\sigma_2\sigma_3\sigma_4\sigma_5 = \sigma_5^{D+1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}$$

$$
\begin{aligned}
&= \sigma_5^A\sigma_4^{-1}\sigma_3^{-1}\sigma_5^{-1}\sigma_4\sigma_5^B\sigma_4^{-1}\sigma_5\sigma_2\sigma_3\sigma_4\sigma_5^C\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1\sigma_5^{-1}\sigma_3\sigma_4\sigma_5\sigma_2\sigma_3\sigma_4 \\
&\quad \sigma_5^{D+1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1\sigma_2\sigma_3\sigma_4^{E+1}\sigma_3\sigma_2\sigma_3\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_5^{F-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1} \\
&\quad \sigma_5^{-1}\sigma_4^{-1}\sigma_3\sigma_4
\end{aligned}
$$

$$\sigma_4^{E+1}\sigma_3\sigma_2\sigma_3\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1} = \sigma_4^E\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_4\sigma_3\sigma_4$$

$$
\begin{aligned}
&= \sigma_5^A\sigma_4^{-1}\sigma_3^{-1}\sigma_5^{-1}\sigma_4\sigma_5^B\sigma_4^{-1}\sigma_5\sigma_2\sigma_3\sigma_4\sigma_5^C\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1\sigma_5^{-1}\sigma_3\sigma_4\sigma_5\sigma_2\sigma_3\sigma_4 \\
&\quad \sigma_5^{D+1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1\sigma_2\sigma_3\sigma_4^E\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_4\sigma_3\sigma_4\sigma_5^{F-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_5^{-1} \\
&\quad \sigma_4^{-1}\sigma_3\sigma_4
\end{aligned}
$$

$$\sigma_5\sigma_2\sigma_3\sigma_4\sigma_5^C\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1\sigma_5^{-1} = \sigma_2\sigma_3\sigma_5\sigma_4\sigma_5^C\sigma_4^{-1}\sigma_5^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1$$

$$
\begin{aligned}
&= \sigma_5^A\sigma_4^{-1}\sigma_3^{-1}\sigma_5^{-1}\sigma_4\sigma_5^B\sigma_4^{-1}\sigma_2\sigma_3\sigma_5\sigma_4\sigma_5^C\sigma_4^{-1}\sigma_5^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1\sigma_3\sigma_4\sigma_5\sigma_2\sigma_3\sigma_4 \\
&\quad \sigma_5^{D+1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1\sigma_2\sigma_3\sigma_4^E\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_4\sigma_3\sigma_4\sigma_5^{F-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_5^{-1} \\
&\quad \sigma_4^{-1}\sigma_3\sigma_4
\end{aligned}
$$

$$\sigma_5\sigma_4\sigma_5^C\sigma_4^{-1} = \sigma_4^C$$

$$
\begin{aligned}
&= \sigma_5^A\sigma_4^{-1}\sigma_3^{-1}\sigma_5^{-1}\sigma_4\sigma_5^B\sigma_4^{-1}\sigma_2\sigma_3\sigma_4^C\sigma_3^{-1}\sigma_2^{-1}\sigma_1\sigma_3\sigma_4\sigma_5\sigma_2\sigma_3\sigma_4\sigma_5^{D+1}\sigma_4^{-1}\sigma_3^{-1} \\
&\quad \sigma_2^{-1}\sigma_1\sigma_2\sigma_3\sigma_4^E\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_4\sigma_3\sigma_4\sigma_5^{F-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_5^{-1}\sigma_4^{-1}\sigma_3\sigma_4
\end{aligned}
$$

$$
\begin{aligned}
&\sigma_5^A\ldots\sigma_5^{F-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_5^{-1}\sigma_4^{-1}\sigma_3\sigma_4 \cong \\
&\quad \sigma_5^{-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_5^{-1}\sigma_4^{-1}\sigma_3\sigma_4\sigma_5^A\ldots\sigma_5^F
\end{aligned}
$$

$$
\begin{aligned}
&\cong \sigma_5^{-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_5^{-1}\sigma_4^{-1}\sigma_3\sigma_4\sigma_5^A\sigma_4^{-1}\sigma_3^{-1}\sigma_5^{-1}\sigma_4\sigma_5^B\sigma_4^{-1}\sigma_2\sigma_3\sigma_4^C\sigma_3^{-1} \\
&\quad \sigma_2^{-1}\sigma_1\sigma_3\sigma_4\sigma_5\sigma_2\sigma_3\sigma_4\sigma_5^{D+1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1\sigma_2\sigma_3\sigma_4^E\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_4\sigma_3\sigma_4\sigma_5^F
\end{aligned}
$$

$$\sigma_5^{-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_5^{-1}\sigma_4^{-1}\sigma_3\sigma_4\sigma_5^A = \sigma_4^{-1}\sigma_3^{-1}\sigma_2\sigma_3\sigma_4^A\sigma_5^{-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}$$

$$
\begin{aligned}
&= \sigma_4^{-1}\sigma_3^{-1}\sigma_2\sigma_3\sigma_4^A\sigma_5^{-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_5^{-1}\sigma_4\sigma_5^B\sigma_4^{-1}\sigma_2\sigma_3\sigma_4^C\sigma_3^{-1} \\
&\quad \sigma_2^{-1}\sigma_1\sigma_3\sigma_4\sigma_5\sigma_2\sigma_3\sigma_4\sigma_5^{D+1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1\sigma_2\sigma_3\sigma_4^E\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_4\sigma_3\sigma_4\sigma_5^F
\end{aligned}
$$

$$
\begin{aligned}
&\sigma_5^{-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_5^{-1}\sigma_4\sigma_5^B\sigma_4^{-1}\sigma_2\sigma_3\sigma_4^C\sigma_3^{-1}\sigma_2^{-1} = \\
&\quad \sigma_3^{-1}\sigma_2^{-1}\sigma_4^{-1}\sigma_3\sigma_4^B\sigma_3^{-1}\sigma_1\sigma_2\sigma_3^C\sigma_2^{-1}\sigma_1^{-1}\sigma_5^{-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}
\end{aligned}
$$

$$
\begin{aligned}
&= \sigma_4^{-1}\sigma_3^{-1}\sigma_2\sigma_3\sigma_4^A\sigma_3^{-1}\sigma_2^{-1}\sigma_4^{-1}\sigma_3\sigma_4^B\sigma_3^{-1}\sigma_1\sigma_2\sigma_3^C\sigma_2^{-1}\sigma_1^{-1}\sigma_5^{-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1} \\
&\quad \sigma_1^{-1}\sigma_1\sigma_3\sigma_4\sigma_5\sigma_2\sigma_3\sigma_4\sigma_5^{D+1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1\sigma_2\sigma_3\sigma_4^E\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_4\sigma_3\sigma_4\sigma_5^F
\end{aligned}
$$

$$\sigma_5^{-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_1\sigma_3\sigma_4\sigma_5 = \sigma_2\sigma_3\sigma_4\sigma_5^{-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}$$

$$= \sigma_4^{-1}\sigma_3^{-1}\sigma_2\sigma_3\sigma_4^A\sigma_3^{-1}\sigma_2^{-1}\sigma_4^{-1}\sigma_3\sigma_4^B\sigma_3^{-1}\sigma_1\sigma_2\sigma_3^C\sigma_2^{-1}\sigma_1^{-1}\sigma_2\sigma_3\sigma_4\sigma_5^{-1}\sigma_4^{-1}$$
$$\sigma_3^{-1}\sigma_2^{-1}\sigma_2\sigma_3\sigma_4\sigma_5^{D+1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1\sigma_2\sigma_3\sigma_4^E\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_4\sigma_3\sigma_4\sigma_5^F$$

$$\sigma_5^{-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_2\sigma_3\sigma_4\sigma_5^{D+1} = \sigma_5^D$$

$$= \sigma_4^{-1}\sigma_3^{-1}\sigma_2\sigma_3\sigma_4^A\sigma_3^{-1}\sigma_2^{-1}\sigma_4^{-1}\sigma_3\sigma_4^B\sigma_3^{-1}\sigma_1\sigma_2\sigma_3^C\sigma_2^{-1}\sigma_1^{-1}\sigma_2\sigma_3\sigma_4\sigma_5^D\sigma_4^{-1}$$
$$\sigma_3^{-1}\sigma_2^{-1}\sigma_1\sigma_2\sigma_3\sigma_4^E\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_4\sigma_3\sigma_4\sigma_5^F$$

$$\sigma_4^{-1}\sigma_3^{-1}\sigma_2\sigma_3\sigma_4^A\sigma_3^{-1}\sigma_2^{-1}\sigma_4^{-1}\dots\sigma_4\sigma_3\sigma_4\sigma_5^F = \sigma_3^{-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2\sigma_3^A\sigma_2^{-1}\dots\sigma_3\sigma_4\sigma_5^F\sigma_3$$

$$= \sigma_3^{-1}\sigma_4^{-1}\sigma_3^{-1}\sigma_2\sigma_3^A\sigma_2^{-1}\sigma_3\sigma_4^B\sigma_3^{-1}\sigma_1\sigma_2\sigma_3^C\sigma_2^{-1}\sigma_1^{-1}\sigma_2\sigma_3\sigma_4\sigma_5^D\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}$$
$$\sigma_1\sigma_2\sigma_3\sigma_4^E\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_3\sigma_4\sigma_5^F\sigma_3$$

$$\sigma_3^{-1}\sigma_4^{-1}\sigma_3^{-1}\dots\sigma_5^F\sigma_3 = \sigma_4^{-1}\sigma_3^{-1}\dots\sigma_5^F$$

$$\cong \sigma_4^{-1}\sigma_3^{-1}\sigma_2\sigma_3^A\sigma_2^{-1}\sigma_3\sigma_4^B\sigma_3^{-1}\sigma_1\sigma_2\sigma_3^C\sigma_2^{-1}\sigma_1^{-1}\sigma_2\sigma_3\sigma_4\sigma_5^D\sigma_4^{-1}\sigma_3^{-1}\sigma_2^{-1}\sigma_1$$
$$\sigma_2\sigma_3\sigma_4^E\sigma_3^{-1}\sigma_2^{-1}\sigma_1^{-1}\sigma_3\sigma_4\sigma_5^F$$

We can simplify our braid word by applying our compressed notation, $\sigma_{i,j}^m$.

$$\cong \sigma_{2,4}^A\sigma_{3,5}^B\sigma_{1,4}^C\sigma_{2,6}^D\sigma_{1,5}^E\sigma_{3,6}^F$$

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